LOGICA 2019

ABSTRACTS

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Making Sense of Relevance Logic

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Relevance logic looks like a failed research program; current accounts all have major shortcomings. However, there is a way in which we can make good intuitive and formal sense of it, without any funny metaphysics and not too complicated. The idea: use semantic decomposition trees (also known as truth-trees, semantic tableaux) essentially as in classical logic, but controlling them for "parity".

I want to leave a lot of time for discussion, so I will take for granted that: you are familiar with the use of such trees in classical propositional logic; have heard of relevance logics and why it was felt by some that they are needed; hopefully, have a vague memory of what a relevance logic such as R looks like in terms of one of its many modes of presentation (natural deduction, hilbertian axiomatization, possible-worlds models with ternary relations, algebraic models, sequents . . .); optimally, have some idea of the shortcomings of these presentations – but don't worry if you have forgotten that.

The talk will work from examples, gradually pointing towards the basic definitions, observations, and open questions.

Logic and Ethics

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The condition under which it is correct (proper) to make an assertion is that the assertor knows how (is able) to perform the task which constitutes the content of the assertion (correctness condition for assertions).

To make an assertion is to commit (obligate) yourself to performing the task which constitutes the content of the assertion (commitment account of assertion).

The condition under which it is correct (proper) to undertake an obligation (make a commitment) is that the obligor knows how (is able) to fulfil it (ought implies can).

The relation between the preceding three principles is simple: the correctness condition for assertions follows from the commitment account of assertion taken together with the ought implies can principle. Both the commitment account of assertion and the ought implies can principle bring in the notion of duty (obligation) and hence implicitly, by the correlativity of rights and duties, the notion of right. On the other hand, the notions of right and duty are the key notions of deontological ethics. Thus, all in all, logic has, not only an ontological layer and an epistemological layer, but also a deontological layer underlying the epistemological one. It can be avoided only by treating the notion of knowledge how (can) as a primitive notion, thereby abstaining from relating it to the notions of right and duty (may and must).

Logical Foundations of Categorization Theory

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Categories are cognitive tools that humans use to organize their experience, understand and function in the world, and understand and interact with each other, by grouping things together which can be meaningfully compared and evaluated. They are key to the use of language, the construction of knowledge and identity, and the formation of agents' evaluations and decisions. Categorization is the basic operation humans perform e.g. when they relate experiences/actions/objects in the present to ones in the past, thereby recognizing them as instances of the same type. This is what we do when we try and understand what an object is or does, or what a situation means, and when we make judgments or decisions based on experience. The literature on categorization is expanding rapidly in fields ranging from cognitive linguistics to social and management science to AI, and the emerging insights common to these disciplines concern the dynamic essence of categories, and the tight interconnection between the dynamics of categories and processes of social interaction. However, these key aspects are precisely those that both the extant foundational views on categorization and the extant mathematical models for concept-formation struggle the most to address. In this talk, I will posit that categorization is the single cognitive mechanism underlying meaning-attribution, value-attribution and decision-making. I will discuss a logical approach which aims at creating an environment in which these three cognitive processes can be analyzed in their relationships to one another, and propose several research directions, developing which, logicians can build novel foundations of categorization theory.

A Typed Term Calculus for Core Logic

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In a series of papers and books spanning four decades, Neil Tennant has developed a logical system first called "intuitionist relevant logic" and more recently "core logic". Core logic is a relative of intuitionistic logic, but is weaker in distinctive ways having to do with its handling of contradictory sets of formulas, and its requirement that all proofs be normal.

Tennant has offered a variety of motivations for core logic, among them claims to its benefits for *computational* approach to logic. In this vein, Tennant has emphasized features of *proof search* in core logic. However, there is another natural way to connect logics to computation, based on the *Curry–Howard correspondence* between proofs and programs, and between proof normalisation and program execution. This presentation develops a typed term calculus that relates to (the implication-negation fragment of) core logic in this way.

Contributed talks

Why it is Okay to Be an Absolutist About Quantification

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According to (Rayo & Uzquiano, 2006, p. 3), *absolutism* is the view that one can engage in inquiry concerning absolutely everything. This is a rather contentious position that has lead to a large volume of literature, in particular, in relation to the problem of Absolute Generality (for a statement of this problem, cf. Linnebo, 2012). In the context of a more restricted version of the problem, where we only want to talk about *all sets*, the thesis of *Absolute Generality in Set Theory* can be taken to be the following:

> *AGS* **In pure set theory, our quantifiers can sensibly be regarded as ranging over absolutely all pure sets**.

In this talk, we will discuss a postulate that we call the principle of *Absolute Infinity of Atoms*, which we can for the moment express as follows:

> *AIA* **No pure set is larger than every actually existing set of urelements**.

Our aim is to use a precise formal version of *AIA* to argue for the coherence of *AGS*.

AIA can be seen as motivated by static modal ontologies such as Lewis's modal realism (Lewis, 1986) and Williamson's necessitism (Williamson, 2013) as both can be shown (under minimal assumptions) to entail the existence of at least as many urelements as sets (see Nolan, 1996; Sider, 2009). However, our purpose is not to defend such ontologies but, rather, to spell out the implications of the consistency of *AIA* for *AGS*.

Cantor recognized that some collections – notably, the collection of all well-order types – are themselves "too big" to be assigned a definite size, a definite cardinal number. It seems it is quite intuitive to say that the "size" of a set that cannot be measured by the cardinals that emerge in the course of the set formation process represents an unsurpassable – albeit mathematically indeterminable – limit, what Cantor called the "absolutely infinite": an "absolute quantitative maximum" that is larger than any set with a definite cardinality and is incapable of either determinable measure or any definite form of increase.

But, what is the connection of *AIA* to *AGS*? A standard challenge to *AGS* is that when making sense of talk about *everything* in a first-order language using model-theoretic orthodoxy, we need to have available a set to represent the universe of quantification, and in standard set theories such a set is not around (Uzquiano, 2006). However, with *AIA* we can provide isomorphic set-like copy of the domain of discourse of all sets. We will argue that this allows us to sensibly talk about all sets in a perfectly consistent manner. A critical premise of our argument is that talking about the objects in a domain D' isomorphic to a purported domain D amounts semantically to the same thing as talking about the objects in D.

We will prove that *AIA* is consistent, when couched in a potentialist theory of sets, relative to ZFC + "There is an inaccessible cardinal". We will take inspiration from (Linnebo, 2013) and develop a potentialist version of the set theory introduced in (Menzel, 2014). The basic thought behind potentialism is to take serious the idea, following Cantor's notion of a set, that any "multiplicity" *can* form a set (Parsons, 1983, p. 315). The modality in this context is understood as a genuine metaphysical modality and not as some sort of special mathematical modality (Linnebo, 2013, pp. 207-8), or a figure of speech.

Specifically, after the pertinent definitions (notably, we define a predicate *Pure* that amounts to saying that a set does not have any atoms in its membership ancestry), we can write *AIA* formally as

AIA $\Box\forall x(Pure(x) \rightarrow \exists y(\forall z(z \in y \leftrightarrow \neg Set(y)) \wedge x \preccurlyeq y))$,

where we define

$$x \preccurlyeq y := \Diamond\exists f(f : x \xrightarrow{1-1} y).$$

So in fact, we are able to accept the intuitions behind the arguments from indefinite extensibility against notions of *set*, *ordinal*, *self-identical object* etc., and simultaneously talk consistently about all these objects because we are able to "represent" them in the structure of the atoms. This provides a way around the argument from indefinite extensibility against *AGS*.

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A Degree-Theoretic Framework for Feasible Knowledge

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The paradox of logical omniscience is a well known flaw of standard epistemic logic. Its implausible conclusion that each epistemic agent knows all tautologies is a consequence of the over-idealization of the agents' deductive powers in the model. In more sophisticated models of epistemic reasoning, logical omniscience can be avoided by considering the agent's *resource-boundedness,* i.e., the fact that performing logical derivations requires some of the agent's limited resources such as time, memory, etc. One of such solutions has been informally sketched by Běhounek (2013); in this approach, resource-awareness is modeled by using suitable semilinear contraction-free substructural logics, also known as *t-norm fuzzy logics* (e.g., Cintula, Hájek, & Noguera, 2011), for epistemic reasoning. A recent formalization of fuzzy intensional semantics (Běhounek & Majer, 2018) makes it possible to elaborate the proposal in more detail.

The proposed solution starts with distinguishing three kinds of knowledge: *actual* knowledge, or the explicit knowledge immediately available to the agent (e.g., written in the agent's database); *potential* knowledge, or the implicit knowledge that is, at least in principle, logically derivable from the actual knowledge; and *feasible* knowledge, or the knowledge that the agent can feasibly derive from the actual knowledge. It can be observed that logical omniscience is only troublesome for feasible knowledge, since actual knowledge need not be closed under logical consequence and potential knowledge does indeed include all logical truths. Feasible knowledge is apparently a *gradual* notion, since long and complex logical derivations require more of the agent's limited resources (e.g., time, memory, or energy) than shorter or simpler ones, and so can be *less feasibly* performed by the agent. The gradual nature of feasible knowledge can be conveniently represented by means of t-norm fuzzy logics, whose truth values are most usually interpreted as degrees of truth (Hájek, 1998). The fact that most t-norm fuzzy logics fall within the class of semilinear contraction-free substructural logics makes them particularly suitable for modeling resource-awareness, since the fusion of resources can be fittingly represented by the multiplicative conjunction $\otimes$ (as is common in linear logic, which is also contraction-free), and the structure of typical resources is semilinear (Běhounek, 2009). The suitability of a particular fuzzy logic for resource-aware reasoning is determined by the intended way of combining the resources: e.g., Gödel–Dummett logic corresponds to maxitive resources (such as erasable memory); Łukasiewicz and product logic, respectively, to bounded and unbounded additive resources (such as computation time); etc.

The proposed framework renders an agent's feasible knowledge as a unary modality K over a suitable propositional fuzzy logic. The truth degree of $\mathrm{K}\varphi$ then represents the degree of feasible derivability of φ by the agent from the actual knowledge, and corresponds directly to the amount of resources needed for the derivation (e.g., the computation time). The modal axioms of standard propositional epistemic logic that express the agent's inference abilities are modified to reflect the cost of derivation; for instance, the resource-aware modification of the axiom (K) of logical rationality reads: $\mathrm{K}\varphi \otimes \mathrm{K}(\varphi \to \psi) \otimes \mathrm{M}_{\varphi,\psi} \to \mathrm{K}\psi$, where the additional propositional constant $\mathrm{M}_{\varphi,\psi}$ represents the cost of applying the rule of modus ponens to φ and $\varphi \to \psi$ by the agent. Similarly the axiom (4) of positive introspection is modified to $\mathrm{K}\varphi \otimes \mathrm{I}_\varphi \to \mathrm{KK}\varphi$, where I_φ reflects the cost of realizing the knowledge of φ. The sub-idempotence of $\otimes$

then decreases appropriately the lower bound ensured by the axioms for the truth degree of $\mathrm{K}\varphi_i$ with each step in the derivation. This eliminates logical omniscience, since the axioms only enforce a small (or even zero) degree of feasible knowability for propositions that require long derivations from the actual knowledge.

Semantically, the described resource-sensitive logic of feasible knowledge can conveniently be modeled in fuzzy intensional semantics (Běhounek & Majer, 2018), where possible worlds represent the agent's epistemic states and transitions between them correspond to inference steps (i.e., changes of the actual knowledge). The fuzzy accessibility relation between states w_1, w_2 represents the cost of performing the inference step from w_1 to w_2. Based on the actual knowledge in each state, the feasible knowledge of a proposition is calculated as a standard (forward-looking possibility-style) fuzzy modality in the fuzzy Kripke frame. Various conditions on the fuzzy accessibility relation (such as fuzzy transitivity, directedness, etc.) and on the evaluations in the fuzzy Kripke model (e.g., persistence, bivalence, etc.) reflect various modal epistemic axioms and yield various levels or real-world plausibility. With appropriate adjustments, the framework admits considering multiple epistemic agents, nesting of the epistemic modalities, and combining freely factual and epistemic subformulae. The agents' potential and actual knowledge can be expressed in the framework as well, namely by setting appropriate thresholds on the feasibility degrees (in the simplest cases, 0 and 1). Additionally, the use of fuzzy logic enables smooth accommodation of gradual propositions in the formalism.

The talk will present the details and features of the proposed apparatus; discuss the plausibility of its assumptions and the resource-sensitive modifications to epistemic principles; and compare it with propositional dynamic logic and related approaches to resource-sensitive epistemic reasoning (e.g., Ho, 1997; Artemov & Kuznets, 2014).

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Frege and Peano on Axiomatisation

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It is commonplace in contemporary historical studies to distinguish two traditions in early mathematical logic: the algebra of logic tradition and the tradition pioneered by Frege. Although he never defended a logicist position, Peano is usually linked to the Fregean tradition. In this talk I shall discuss this association. Specifically, I shall study Frege's and Peano's conceptions of axiomatisation and analyse their evolution. I shall defend, on the one hand, that Peano's early notion of calculus was influenced by Schröder and, on the other, that Peano's later development of a calculus of logic can be seen as a departure from the algebra of logic tradition.

Frege presented the first modern deductive system in (1879). In this work he developed a logical calculus that could be seen as a theory of inference. Frege's stress on a systematisation of the notion of inference was further developed in (1893), where he presented a refined version of his formal system, the concept-script. Frege's notion of calculus was connected with the Euclidean tradition of an axiomatic method, but it also involved a deductive aspect. The later calculus of the concept-script was composed of a set of logical principles, the basic laws, and a set of inference rules. Accordingly, the proofs of the concept-script were completely regimented by inference rules, in such a way that no formal step was left implicit and no appeal to non-defined logical principles needed to be made.

Peano provided the first axiomatisation of arithmetic in (1889). His approach fits with the trend in nineteenth-century mathematics of arithmetisation of analysis. As a means of relieving arithmetic of the use of natural language, Peano complemented the basic linguistic elements of arithmetic with the formal resources provided by logic.

Peano used as a basis for his mathematical logic the calculus of classes and the calculus of propositions developed by algebraic logicians. However, unlike them, Peano was reluctant to use arithmetical symbols in order to express logical or set-theoretic relations. He wanted to preserve the specific meaning of arithmetical symbols and, at the same time, avoid the confusions that would arise had they acquired, in addition to their mathematical meaning, a logical meaning. In fact, one of the most significant elements in Peano's axiomatisation of arithmetic is the clear separation of logical principles and arithmetical principles[1]. In this regard, he departed from the algebra of logic tradition.

Most likely because of the development of his logicist project, Frege (1897) failed to acknowledge the importance of Peano's axiomatisation. Nevertheless, he praised the expressive capabilities of Peano's logical language and contrasted it with Boole's and Schröder's – which, from Frege's perspective, were inadequate as tools for the expression of arithmetical truths. At the same time, Frege criticised Peano's failure to provide a deductive calculus; as a matter of fact, in his early writings on logic Peano did not define any inference rule. According to Frege, Peano could not guarantee a fully rigorous treatment of arithmetic if he did not provide the means to formalise proofs. After all, in his early works Peano understood the principles of reasoning as rules for transformation of formulas and identified them with logical identities.

[1] Nevertheless, in (1889) Peano did not distinguish the axioms of the calculus of classes and the calculus of propositions from the theorems. In this work, the principles of logic were listed without any distinction concerning their status in the calculus.

In contrast, Frege made explicit all the logical laws used in the deduction of a logical theorem and regimented this proof as a series of formal steps that consisted in the application of an inference rule.

Peano departed from an algebraic theory of logic, Schröder's algebra of logic, that was not intended for use as a deductive system. Algebraic logicians developed this algebraic theory by means of the same reasoning that they used for the development of mathematical theories. They thus did not perceive the convenience of establishing logical laws of reasoning and isolating a set of inference rules. However, Peano was not interested in studying the mathematical principles of logic, as Schröder did. He disregarded algebraic principles in constructing an axiomatic system of logic. Yet since he developed his logic of classes and sentential logic upon Boole's, Peirce's and Schröder's calculi of classes and calculi of propositions, he was not pressed to acknowledge that the formalisation of logical reasoning – which was instrumental for obtaining arithmetical theorems – required, besides a specific set of primitive propositions, a set of inference rules. I defend that the fact that he relied on the algebra of logic as a basis for his mathematical logic made it difficult for him to perceive the shortcomings of his calculus as a deductive calculus. As a support for this claim, I evaluate Peano's decision to conflate the conditional and the relation of logical consequence.

Although Frege had criticised in (1897) both the ambiguity of Peano's conditional symbol and the lack of inference rules in Peano's mathematical logic, Peano did not deal with these issues in his answer to Frege (1898). Nevertheless, Peano partially modified his position in later works. In (1899) he highlighted the specific nature of the relation of logical consequence, although he still expressed it using the conditional symbol. At the same time, Peano modified his presentation of the rules of reasoning; from (1899) on, the inference rules were listed, distinguished from logical propositions and presented schematically. The proofs contained in (1900) witness Peano's development of a deductive calculus of logic.

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New Science of Infinity

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1. Cantor established two kinds of infinity: cardinal and ordinal numbers, each with its own arithmetic and its own relation *greater than*. In modern developments, ordinal numbers are special sets, cardinal numbers are specific ordinal numbers. In both cases, the set of natural numbers $\mathbb{N}$ makes the yardstick of infinity, be it the cardinal number $\aleph_0$ or the ordinal ω. In this way, Cantor's theory of infinite numbers assumes that finite numbers are positive integer, and it seeks to extend the system $(\mathbb{N},+,\cdot,0,1,<)$. However, while Cantor's infinities try to extend the system of finite numbers, they hardly mimic its arithmetic, e.g. the addition and multiplication of ordinal numbers are not commutative, and the order is not compatible with the arithmetic operations.

2. (Gödel, 1947) presents Cantor's infinite numbers as extending the system of natural numbers $(\mathbb{N},+,\cdot,0,1,<)$, and seeks to show that "this extension can be effected in a uniquely determined manner". To this end Gödel discusses (1) definition of infinite numbers, (2) their equality, (3) total order, and (4) operations of sum and product.

(Ad 1) Gödel implicitly assumes that the very definition of infinite number has to be derived from the notion of infinite set. (Ad $2,3$) He also claims that "there is hardly any choice left but to accept Cantor's definition of equality between numbers, which can easily be extended to a definition of 'greater' and 'less' for infinite numbers". Here, the equality is based upon the one-to-one correspondence, while the total order of infinite numbers *extends* what Cantor called *the natural* order of integers. As there are many possible well-orderings on the set $\mathbb{N}$, Cantor considered the *natural* one. Similarly, in (Cantor, 1895) when he studied the order type θ of real numbers, he also mentioned the *natural* order of the real numbers. Cantor, however, could never explain what does *the natural* order mean in mathematical terms, be it the order of $\mathbb{N}$ or $\mathbb{R}$. In fact, his hierarchy of ordinal numbers is based on the notion of well-ordering with no reference to algebraic operations on finite numbers. (Ad 4) Finally, Gödel writes: "it becomes possible to extend (again without any arbitrariness) the arithmetical operations to infinite numbers (including sums and products with any infinite number of terms or factors) and to prove practically all ordinary rules of computation". Arguably, *practically all rules* does not include commutativity of sum and product, and compatibility of sum and product with the order.

3. Cantor's set theory itself provides grounds for an alternative arithmetics for ordinal numbers, namely Cantor's theorem on normal form of ordinal number introduced in (Cantor, 1897) enables an alternative definition of sum and product. It follows from this theorem that every ordinal number $\alpha > 0$ has the unique representation in a finite (*polynomial*) form $\alpha = \omega^{\eta_1}\cdot p_1 + \ldots + \omega^{\eta_h}\cdot p_h$, where $\eta_1 > \ldots > \eta_h$, $\eta_i \in Ord$, h, $p_i, q_i \in \mathbb{N}$, $1 \leq i \leq h$. Based on this representation, the so called *normal* sum and *normal* product of ordinal numbers are defined in way similar to the sum and product of polynomials; see (Hessenberg, 1906), (Kuratowski & Mostowski, 1976). While these new operations are commutative and compatible with the order of ordinal numbers, we can claim that ordinal numbers with these *normal* operations and the *standard* total order apply to all the rules of the arithmetic of finite numbers. Still, our objectives go far beyond that. We present an ordered field that includes the class of ordinal numbers *Ord* with the *normal* operations and the *standard* total order. Yet, our starting point is an alternative interpretation of a finite number.

4. The perspective adopted is that finite number is a real number. By extending the system $(\mathbb{R},+,\cdot,0,1,<)$, we obtain a non-Archimedean field that necessarily includes infinitesimals. Accordingly, we define infinite numbers as inverses of infinitesimals. The *biggest* non-Archimedean field is the field of surreal numbers as developed in (Conway, 1976), (Conway, 2001), and (Goshor, 1986). We show that it includes Cantor's ordinal numbers, though with their *normal* sum and *normal* product. Thus, in our theory, Cantor's infinite numbers as well as infinitesimals belong to one and the same mathematical system of the commutative ordered field. As a consequence, in addition to the number ω, that system also includes numbers like $\frac{\omega}{2}$, $\frac{1}{\omega}$, $-\omega$, as well as $\sqrt{\omega}$ (since the field of surreal numbers is a real closed field). Similarly, within that system, each Cantor's ordinal number is subject to ordered field operations.

5. We show that our specific understanding of finiteness originates in Euclid's notion of $\mu\varepsilon\gamma\varepsilon\theta o\varsigma$. Then, *via* a field of line segments as developed in (Descartes, 1637), it evolved into a non-Archimedean field explored in (Euler, 1748), and (Euler, 1755). In fact, Euler explicitly defined infinite numbers as inverses of infinitesimals. On the other hand, Cantor repeatedly sought to demonstrate the inconsistency of infinitesimals; see for example (Cantor, 1887). Within our framework, we can easily demonstrate flaws in his arguments.

6. General accounts of mathematical infinity, such as (Moore, 1990) or (Heller & Woodin, 2011), suffer from a puzzling dichotomy. They develop their analysis, on the one hand, in the context of calculus, in conjunction with the definition of the limit of sequence; on the other hand, in the context of set theory, in regard with cardinal and ordinal numbers. Significantly, this duality is not commented on, and most importantly, it is not even noticed at all. Focusing on (Euler, 1748), we will provide a unified account of infinity, one that combines infinite numbers with a version of calculus, namely, the nonstandard analysis.

7. Standard interpretations of infinitesimals put them in the context of extending the line of real numbers. Thus, infinitely small and infinitely large numbers are discussed in a philosophical context as an alternative view of continuum, or in a mathematical context, as an alternative to the epsilon-delta technique. In our development, they are also discussed as an alternative to Cantor's account of infinity.

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Dialogical Criteria of Adequate Formalisation

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According to Peregrin and Svoboda's recent proposal, adequacy criteria for logical formalisations should not be based on semantic or syntactic considerations regarding the isolated natural language sentence and its corresponding formalisation. Instead, they argue for the idea of studying the sentence's inferential behaviour - and that of its formalisation - in a certain number of sample arguments in which the sentence and its formalisation occur as premise or as conclusion. Hence, they develop two principles which together aim at identifying the more adequate formalisation by evaluating the correspondence between the intuitive (in-)correctness of the sample arguments in question and the logical (in-)validity of the diverging argument forms revealing the alternative ways to formalise the same natural language sentence. We are thus confronted with *inferential* criteria of adequacy which take a *holistic* perspective on the question of formalisation.

However, the resulting theory of formalisation fails to meet the underlying requirements of inferentialism and holism and therefore undermines the project's initial ambitions. First, the theory is not entirely *inferential* because Peregrin and Svoboda were forced to introduce some non-inferential auxiliary criteria in order to respond to the lack of syntactic sensitivity of their primary inferential criteria. Second, the theory is not *holistic* because the criteria are only formulated for and applied to formalisations of single sentences whose behaviour is to be tested in a set of sample arguments for which the adequacy of the remaining formalisations that make up a given argument is taken for granted.

The aim of this paper is thus to restore the inferential and holistic features which are at the bottom of Peregrin and Svoboda's project. I will do so by examining the consequences of applying current developments in the field of dialogical logic to the problem of formalisation. Since the dialogical approach shares some of the most distinctive assumptions of Peregrin and Svoboda's conception of semantic inferentialism and the normative role of logic, I will claim that a dialogical reformulation of their principles yields a fruitful modification of their approach to the theory of formalisation which can avoid the problems mentioned above while staying entirely consistent with their overall perspective. Hence, my present investigation on the yet unexplored potential of what the dialogical approach can contribute to the question of formalisation will amount to the development of genuinely inferential and holistic criteria of adequacy. I thus argue that the idea of dialogical adequacy criteria, which has so far been neglected by both formalisation and dialogical theorists, will reveal itself as a powerful tool for adequately formalising natural language arguments while at the same time extend the realm of application of the dialogical approach to logic.

After a brief survey of Peregrin and Svoboda's take on the adequacy of formalisation, I will first address the problem concerning holism by following Reinmuth's recent suggestion of shifting the perspective from the assessment of alternatively formalised *single* sentences within a rigid frame of seemingly unproblematic formalisations to the evaluation of the adequacy of *entire sets* of formalisations relative to a given natural language argument. I will then provide a short introduction to the field of dialogical logic by drawing on the most recent developments made by Rahman and colleagues. By incorporating some features of Martin-Löf's

Constructive Type Theory into the dialogical approach as the possibility of providing both *local* and *strategic reasons* for the propositions brought forward in a given dialogue, they constructed a logical framework for immanent reasoning which is closely related to Brandom's *game of giving and asking for reasons*. In other words, by developing a logical structure in which meaning and justification are both constituted in the object language, i.e. by rule-governed argumentative interaction only, the dialogical conception of logic can now reveal explicit instructions on how to argue for and against a given thesis.

After my introduction to dialogical logic, I will then be able to turn to the actual elaboration of dialogical criteria of adequate formalisation. The idea is that, on the formal side, the dialogical games for *alternative* formalisations of the *same* argument reveal diverging dependences and distributions of challenges and defences. By drawing on Brandom's *model of assertional practice*, I will then propose a procedure for developing an informal argumentative strategy on the corresponding natural language argument which makes explicit the conditions to support the argument in terms of the reasons that are given and asked for. Consequently, the formalisation, whose dependence and distribution of *duties to defend* and *rights to challenge* in the formal dialogue correspond more closely to the pattern of *duties to give* and *rights to ask for reasons* in the natural language argumentative strategy, will be qualified as being the *more* adequate formalisation.

But before comparing this degree of correspondence, it must first be assured that a valid argument form will not yield instances of intuitively incorrect natural language arguments and vice versa. This essential requirement of a bilateral correspondence between intuitively (in-) correct arguments and (in-)valid argument forms is what Peregrin and Svoboda expressed by means of their two inferential criteria of adequate formalisation. At this point, I will thus develop a dialogical reformulation of the *Principles of Reliability* and *Ambitiousness* which will be formulated in terms of winning-strategies for the player defending the thesis, i.e. for the *Proponent*. Only if a given set of formalisations passes the test of *Reliability* and of *Ambitiousness*, it can be conceived of as *an* adequate formalisation. However, if it turns out that there is more than one set of *adequate* formalisations of the *same* set of natural language sentences, then a further criterion is required. But instead of falling back to some non-inferential auxiliary criteria, I will develop a third dialogical, and thus inferential, *Principle of Correspondence*.

As mentioned above, this principle will compare the degree of correspondence between the varying distribution of *duties to defend* and *rights to challenge* emerging from the dialogical games for each of the alternative sets of (adequate) formalisations with the pattern of *duties to give* and *rights to ask for reasons* in the informal argumentative strategy. Consequently, it will be possible to identify the *more* adequate formalisation by means of a purely inferential method of justification which will turn out to be highly sensitive to even the slightest syntactic variations between the remaining formalisation candidates.

In the last section, I will then apply the dialogical criteria of adequate formalisation to Peregrin and Svoboda's example *No grey donkeys are lazy*. Having rejected their reliance on a 'ready-made' formalisation of the inferential frame, I will slightly modify one of the alternative sets of formalisations and, more importantly, adjust the example's original formalisations by reference to Ranta's *Type-Theoretical Grammar* so that they become viable in a context of dialogical games for immanent reasoning. The example will then allow me to illustrate the conceptual advantages of substituting the non-holistic and only partly inferential criteria of the original approach with the dialogical principles developed above. To conclude, I will raise some concerns and suggest directions for future research.

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Open-ended Quantification and Non-standard Models

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Second-order quantifiers (SOQs) have both a *standard semantics*, in which they range over the entire power set of the domain of discourse, and a Henkin semantics (*general semantics*), in which they range over some collection of subsets of the domain that it is not necessarily the entire power set of that domain. The Henkin models for these quantifiers contain, besides the interpretation function and the first-order non-empty domain of objects (D_1), a certain number of non-empty domains ($D_2, D_3, \ldots$) of *n-ary* relations over D_1, where these additional domains constitute the range of the relational variables of the appropriate kind.

[Antonelli 2013: 637] pointed out that there is an asymmetry in Henkin's approach between the way in which the first-order quantifiers (FOQs) are interpreted in general semantics and the way in which the SOQs are interpreted in the very same semantics. In particular, he emphasizes that there is an implicit assumption in Henkin's approach that the meanings of the FOQs are determined only relative to the entire domain of objects D_1 (more specifically, it is assumed that they range over the entire power of D_1), whereas for determining the meanings of the SOQs it is necessary to consider in addition multiple second-order domains ($D_2, D_3, \ldots$). That his assumption is problematic becomes transparent if we consider Mostowski's theory of generalized quantifiers which takes the FOQs as predicates over the full power set of the domain of objects (D_1). In this approach, the existential first-order quantifier denotes the set of all non-empty subsets of D_1 while the universal first-order quantifier denotes the entire domain D_1.

On the assumption that FOQs are second-order predicates, Antonelli extends Henkin's general models also for the FOQs in order to restore the symmetry between the interpretations of the FOQs and the SOQs. What is thus obtained are what he calls *general models* (i.e., non-standard models) for the FOQs, i.e., models that have, besides the specific interpretation function and the non-empty domain of objects D_1, a second-order domain D_2 of non-empty subsets of D_1. In the *general models* for the FOQs, these quantifiers can be interpreted as ranging not automatically over the full power set of the domain of objects D_1, but rather over some collection of subsets of this domain. In particular, the existential FOQ ranges over a smaller class of non-empty subsets than the entire class of such subsets, while the universal FOQ can range over a subset of D_1 which is smaller than the singleton $\{D_1\}$.

[Bonnay and Westerståhl 2016] used Antonelli's *general models* to provide a characterization of the non-standard models for the FOQs discovered by [Carnap 1937, 1943]. *Carnap's Problem* for a system of logic is defined by them as the question whether there are interpretations which are consistent with the relation of logical consequence ($\vDash$) from that system, but which provide the logical terms with different meanings than the standard ones. The general models defined by Antonelli are non-standard, but not all of them are consistent with the relation of logical consequence in first-order logic since in these general models some first-order validities do not hold (see [Antonelli 2013: 653]). Nevertheless, [Bonnay and Westerståhl 2016: 734-36] proved that if in Antonelli's general models D2 is taken to be a principal filter on D_1, then the resulting interpretations are consistent with $\vDash$. Certainly, the standard interpretation $\{D_1\}$ for $\forall$ is among the consistent interpretations with $\vDash$, but there are many principal filters which are different from the principal filter $\{D_1\}$ and, consequently, there are many non-standard interpretations for $\forall$. More precisely, in these non-standard models, the universal quantifier will not simply mean "all" but will mean "all A", where A is a non-empty set of objects included in the domain D_1 (likewise for the existential quantifier). The main idea is that in a non-standard model the domain D_1 of interpretation is divided in a subset A of D_1, generated by the principal filter, and the subset D_1 A. Only the objects of D_1 that are named in the language are the *real objects* (like in free logic) and belong to A, which will be the proper domain of quantification (i.e., the inner domain).

I show in my presentation that the characterization of the non-standard models for the FOQs provided by [Bonnay and Westerståhl 2016] leaves aside an important kind of non-standard model for these quantifiers, precisely the non-normal interpretation that [Carnap 1937: 232, 1943: 140] described. In this *non-normal interpretation*, a universally quantified sentence "$(\forall x)Fx$" is interpreted as "every individual is F, and b is G", where "b" is an individual constant. The problem here is not that the universal does not range over the entire D_1 – since [Carnap 1943: 136] worked under the assumptions that D_1 is denumerable and every member of it has a name in the language –, but that it is not *logically equivalent* with the conjunction of all its instances (the elimination rule of the universal only guarantees that the universal implies all its instances, but not the converse), and this is why Carnap introduced transfinite rules to provide a unique standard meaning for the FOQs.

The existence of this non-normal interpretation will allow me next to show that even if the formal rules of deduction for the quantifiers are taken to be open-ended, as [McGee 2000, 2006] argued, they still fail in uniquely determine the meanings of the quantifiers. [McGee 2000, 2015] proposed an open-ended logical inferentialism programme and argued for the idea that if the formal natural deduction rules of first-order logic are taken to be open-ended (i.e., they continue to hold even if the language expands), then they uniquely determine the meanings of the logical terms that they introduce and, consequently, the universal quantifier should be taken as ranging over absolutely everything, rather than over a subset of the universal set. Certainly, this would imply, against the Löwenheim-Skolem theorem, that all models of a countable first order theory with an infinite model have the same cardinality – that of the universal set. What I show is that McGee's open-endedness requirement only succeeds to uniquely determine the cardinality of the domain of quantification (provided that we accept the assumption that every object is nameable), but it does not succeed in uniquely determining the meanings of the quantifiers.

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Inferences and Metainferences in ST

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The logic ST has been proposed to deal with paradoxes of vagueness and with the semantic paradoxes (Cobreros et al. (2012), Cobreros et al. (2013), Ripley (2013)). There is something very distinctive about ST: namely, it is classical logic for a classical language, but it provides ways of *strengthening* classical logic to deal with paradoxes. For example, the logic ST^+ (ST for a language with a transparent truth predicate and self-referential sentences) is a *conservative extension* of classical logic. That is, ST^+ is not only non-trivial, but it has exactly the same valid inferences as classical logic for the T-free fragment. How is this possible? Well, because ST^+ preserves all classically valid *inferences* but not some classical *metainferences*. The question then arises of exactly which are the metainferences of ST^+. In a 2015 article, Eduardo Barrio, Lucas Rosenblatt and Diego Tajer give a precise answer to this question.

A **metainference** is a conditional statement of the following form:

$$\text{(MI)} \quad \Gamma_1 \vdash \Delta_1; \ldots \Gamma_n \vdash \Delta_n \implies \Gamma_{-1} \vdash \Delta_{-1}; \ldots \Gamma_{-k} \vdash \Delta_{-k}$$

Where the Γ's and Δ's are sets of formulas of a (suitable) propositional language with liar-like sentences. An ST^+**-instance** of a metainference is a uniform substitution of propositional letters in the metainference by formulas of the language and all turnstile symbols by the double turnstile symbol ST^+ superscripted. We will say that **a metainference holds for** ST^+ when all its ST^+ instances are true. For example,

$$A \vdash B \wedge \neg B \quad \implies \quad A \vdash C$$

is a metainference and,

$$\lambda \vDash^{\mathsf{ST}^+} A \wedge \neg A \quad \implies \quad \lambda \vDash^{\mathsf{ST}^+} \bot,$$

is an ST^+-instance of it. This particular instance is true since it is true that $\lambda \vDash^{\mathsf{ST}^+} \bot$. The metainference, however, does not hold for ST^+ since the following is a false instance:

$$\top \vDash^{\mathsf{ST}^+} \lambda \wedge \neg\lambda \quad \text{BUT} \quad \top \nvDash^{\mathsf{ST}^+} \bot$$

The metainference in question has the flavor of a failure of explosion. In fact, Barrio et al. (2015) show that there is a close connection between metainferences in ST^+ and inferences in Priest's Logic of Pardox LP:

$$\text{The metainference} \quad \Gamma \vdash \Delta \implies \Gamma' \vdash \Delta' \quad \text{holds for } \mathsf{ST}^+$$
$$\text{if and only if}$$
$$\bigwedge\Gamma \supset \bigvee\Delta \vDash^{\mathsf{LP}} \bigwedge\Gamma' \supset \bigvee\Delta'$$

In addition to this semantic connection between metainferences in ST^+ and inferences in LP, Barrio, Rosenblatt and Tajer shows a second, proof-theoretic, connection between ST^+ and LP. The authors argue that ST^+'s *external logic* (from Avron (1988)) must be defined in a particular way and then show that, thus defined, ST^+'s external logic coincides with LP (once again).

In addition to the valuable achievement of finding a way to characterize exactly which inferences hold for ST^+ Barrio, Rosenblatt and Tajer claim that adopting ST^+ as a solution to the paradoxes is no more illuminating than adopting LP (in form of a slogan: ST^+ is LP in sheep's clothes).

In this talk we would like to provide a response to the philosophical claim by reviewing the result in a broader context. In the first place, considering anti-inferences (and meta-anti-inferences), the following is a corollary of the previous proposition,

$$\text{The meta-anti-inference } \Gamma \nvDash \Delta \implies \Gamma' \nvDash \Delta' \text{ holds for } \mathsf{ST}^+$$
$$\text{if and only}$$
$$\bigwedge\Gamma \wedge \neg\bigvee\Delta \vDash^{\mathsf{K3}} \bigwedge\Gamma' \wedge \neg\bigvee\Delta'.$$

About ST's external logic. External validity for a logic X is supposed to tell us something relevant about X, that is, about X's inferences. The following constraint, then, is natural: that anything deemed equivalent by X is also treated as equivalent by X's external logic.

$$\begin{array}{ccc} \Gamma \vdash^{\mathsf{X}} \Delta & & \Gamma \vdash^{\mathsf{X}}_{\mathsf{e}} \Delta \\ \Updownarrow & \Longrightarrow & \Updownarrow \\ \Gamma' \vdash^{\mathsf{X}} \Delta' & & \Gamma' \vdash^{\mathsf{X}}_{\mathsf{e}} \Delta' \end{array}$$

Figure 1: $\mathsf{e}-\mathsf{constraint}$

If this constraint is right, then LP cannot be *the* external logic of ST since ST is self-dual while LP is not. If LP has any claim to be ST's external logic, so has $\mathsf{K3}$ (which is LP's dual). These considerations allows us to conclude that (against the 'LP in sheep's clothes objection') ST is the golden mean between LP and $\mathsf{K3}$.

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Logical Truth Without Truth

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Some of the most basic notions of logic are the notions of logical truth and logical consequence. In modern logic, the widely accepted analysis of classical logical truth and consequence for first-order languages was given by (Tarski, 1936). The nowadays most commonly used definitions of logical truth and consequence are the model-theoretic definitions by (Tarski & Vaught, 1956). Central to these definitions is the possibility to vary the interpretation of the non-logical constants of a sentence over domains of certain models. This presupposes the use of a considerable amount of set theory.

More innocent accounts (in terms of ontology) of logical truth and consequence, which aimed to define these notions purely by means of uniform substitution of linguistic entities were, all the same claimed to be deficient. The reason for that can be seen in the assumption (see also (Tarski, 1936)) that no language could be rich enough in modes of expression to guarantee that if every sentence φ, that can be obtained by uniform substitution of expressions for expressions of the same grammatical category, is true, then φ is true for all set-theoretical interpretations. The apparent problem, as (Quine, 1970) points out, is that "it has been an accepted tenet of classical set theory from Cantor onward that the classes [...] outrun the expressions [of any language]".

Being in favor of avoiding the ontological commitments of set theory, Quine himself gave a substitutional definition of logical truth in (Quine, 1970), which comes out to be extensional equivalent to the model-theoretic definition for first-order languages L without equality, as long as L is strong enough to express elementary arithmetic. However, Quine's definition is also af-flicted with problems, as later research revealed (for a summary see (Ebbs & Goldfarb, 2018)).

In this paper I will consider a substitutional definition of logical truth and consequence that extends Quine's attempt in 4 important points: (1) The given substitutional definition is extensional equivalent to the model-theoretic definition for *any* first-order language L without equality. (2) The substitutional definition of logical consequence in terms of satisfaction is compact, unlike Quine's definition (as observed by (Boolos, 1975)). (3) The given definition is able to eliminate*all* use of model theory, whereas Quine's definition needed at least to employ the standard model of arithmetic. (4) When Quine's definition presupposes the availability of a notion of truth for a language, the presented definition is applicable to any formal language L independent of whether we know how to determine, which sentence of L is true or false.

The central notion of this definition will be that of a translation-function as defined in (Tarski et al., 1953). These translation-functions are used to generate substitutions of predicate symbols in a sentence φ of a language L by suitable formulas in the language of Peano Arithmetic. Instead of considering the truth of any of such substitution instances of φ for determining the logical truth of φ, we will consider the provability of these instances in an extension of Peano Arithmetic. To prove (1) and (2) it will be necessary to make little use of the theory of interpretability. For instance, we extend a theorem of (Feferman, 1960) and prove a formalized version of the completeness theorem:

Theorem 1. *Let Γ be a countable set of sentences of a relational language L of first-order logic without equality s.t. the deductive closure of Γ is arithmetical definable by some $L[\mathrm{PA}]$-formula γ in Σ^0_n. If Γ is satisfiable, then there exists a direct translation f from L to $L[\mathrm{PA}]$ s.t.* $\forall\psi \in \Gamma : \mathrm{PA} + \mathrm{Tr}(\Pi^0_n) \vdash f(\psi))$.

For the presented definition of logical consequence ($LConseq(\Gamma, \varphi)$), it will be an easy corollary of Theorem 1, that:

Theorem 2. *Let φ be any sentence of a relational language L of first-order logic without equality and Γ an arithmetical definable set of L-sentences, then* $\Gamma \models \varphi \Leftrightarrow LConseq(\Gamma, \varphi)$.

In the next step, we will compare the presented definitions of logical truth and consequence to a competing definition given recently by (Halbach, 2019) and discuss its alleged advantages. We will see, that Halbach's definition is on a par with ours in respect to the points (1), (2) and (3). They diverge only when it comes to (4). Halbach's substitutional account requires a primitive notion of satisfaction that is to be axiomatized in a suitable strong theory. The price that has to be paid for this is the rejection of semantic reductionism, i.e. that satisfaction cannot be eliminated by purely mathematical concepts such as set membership or provability in arithmetic (as suggested by our definition).

Finally, we will sketch an idea of how to make our substitutional definition of logical conse-quence also work for first-order logic including equality.

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Fuzzy Semantics for Graded Predicates

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Some predicates apply to the world in degrees. For instance, *warm*, *bent* and *acute* all appear to be graded in this sense, as can be seen from the fact that they can figure in statements such as *Last Winter was the warmest ever recorded*, *The rod is slightly bent*, *A 30° angle is more acute than a 60° one*. Graded predicates have been the subject matter of important debates in three different, although overlapping, disciplines: philosophy, linguistics and mathematical logic. Unsurprisingly, each of these takes a different approach to the matter.

Philosophers mainly tackle foundational (e.g. epistemological and ontological) issues by focussing on a specific sort of graded predicates: vague predicates. By contrast, linguists place less weight on foundational questions and instead try to describe the meaning of as many linguistic constructions as possible involving, in turn, as many different kinds of graded predicates as possible. Finally, mathematical logicians are often motivated both by linguistic and philosophical questions, but ultimately tend to be interested in technical results for their own sake. This leads to a wealth of sophisticated formal tools whose full potential in linguistics and philosophy is, unfortunately, often left unexplored.

As with any interdisciplinary research topic, communication between different approaches appears key to advancing knowledge on the matter. In this case, there has been some fruitful cross-discipline work. On the one hand, an important meeting point between philosophy and linguistics seems to be the contextualist approach to vagueness, according to which shifts in the context of utterance are crucial to resolving Sorites-style paradoxes. For example, Graff (2000) argues for contextualism from a philosophical perspective and Kennedy (2007) adopts a version of her proposal in his semantic analysis. On the other hand, we find substantive collaboration between philosophers and logicians, since many philosophers have taken logic as the source of the puzzles regarding vagueness and have looked at non-classical logics for a solution. For instance, some philosophical theories of vagueness have relied on the use of many-valued logics; in particular, Smith (2008) has developed a philosophical theory of vagueness which helps itself to mathematical fuzzy logic.

Despite the previous efforts to build bridges between the different communities, we believe there is room for more. In particular, the link between mathematical logic and linguistic semantics should be further explored. In this talk, we aim to do that by sketching a way in which linguistic semantics could benefit from recent developments in mathematical fuzzy logic. Thus we adopt the aims of the linguists (and consider the wealth of data they have gathered) while capitalising on the tools of the fuzzy logician.

The very question of how graded predicates should be categorised is an interesting one. To give a sense of the complexity of the phenomenon at hand, let us sketch what we take to be the best taxonomy of graded predicates.[1] Firstly, we distinguish predicates whose applicability can be measured from those (if there are any) whose applicability is not measurable. Uncontroversially, predicates denoting physical qualities fall among the former. Controversially,

[1] Our categorisation stems, in part, from Paoli (1999) and Kennedy and McNally (2002).

perhaps moral and aesthetic adjectives fall among the latter. Amongst the measurable ones, we distinguish uni- from multi-dimensional ones. The former are those whose degree of applicability varies along a single scale; e.g. *tall* (scale: linear extent). By contrast, the latter have various underlying scales; e.g. *intelligent* (scales: memory, arithmetical skill, etc). Among both uni- and multi-dimensional predicates we draw distinctions according to the features of their underlying scales. In particular, we distinguish linear from non-linear scales. Although this is controversial, an example of a uni-dimensional non-linear predicate could be *painful* – not all painful events are comparable and yet there seems to be a single scale of pain. Turning to linear predicates, we distinguish between vague and precise predicates. Vague predicates are characterised (among others) by having blurry boundaries. We have already seen some: *tall* and *warm*. Precise predicates draw sharp divides between their extensions and their anti-extensions but still rely on a scale of degrees for their applicability. Among these we find, as a limiting case, bivalent predicates (e.g. *even number*), but also non-bivalent ones. In turn, the latter are divided into at least three sorts: predicates which demand to reach a maximum amount of a certain quality for their applicability (e.g. *full*), those which instead demand to surpass a minimum amount (e.g. *dirty*) and those whose turning point is neither the minimum nor the maximum of the scale (e.g. *acute angle*). Moreover, we leave the door open to a category of bivalent predicates which have underlying multigraded scales (e.g. *fail an exam*).

After decades of being ruled out due to objections such as that of artificial precision, the fuzzy approach to vagueness was reexamined and vindicated in the form of fuzzy plurivaluationism (Smith, 2008). This approach takes, instead of a single fuzzy model, a set of various fuzzy models for the semantics of a vague predicate (thereby overcoming the artifical precision problem). While this appears to have revived the interest in fuzzy logic as a tool for vagueness to some extent, its potential in formal semantics remains virtually unexplored. In this talk, we take Smith's reformulation of the fuzzy theory of vagueness as our starting point and try to make adjustments to turn it into a more complete semantic theory.

Interestingly, Kennedy's and other linguistic accounts of graded predicates make use of degrees. This suggests a connection with the fuzzy approach, but the former is ultimately classical: the (classical) truth-value of a statement involving a graded predicate is evaluated on a scale of degrees and is based on a contextually given degree which acts as a standard of comparison (e.g. being tall is having the quality of tallness to a degree higher than or equal to the standard of comparison). One of the problems of this semantics is that in order to analyse the meaning of the predicative (unmarked) position, it needs to make use of a null degree morpheme, which serves to transform the measure function denoted by the bare adjective into a classical property. This, although not problematic in itself, in this case receives no independent justification and thus appears to be a device introduced to fix up the semantics. Other things being equal, it would be preferable to make do without this use of a null morpheme.

We propose an alternative account which overcomes this problem, since it takes adjectives to be predicates themselves, albeit fuzzy predicates. In particular, we will make use of very general algebras of truth degrees (defined by uninorms, thus possibly with many designated elements for definitive truth) and we will take all truth scales associated with each kind of adjective to be different kinds of subalgebras thereof. This, as we will argue, will enable us to reduce all facts about gradedness (and, in particular, vagueness) to facts about the forms of the relevant scales.

We will finish our talk by responding to an old objection to the fuzzy approach to vagueness, the objection of Non-Borderline Comparatives (see Paoli (1999)). This will serve as an example of an objection which was raised having in mind one particular structure of fuzzy truth degrees and which, in light of recent developments in the study of fuzzy logics, needs to be revisited today.

Our conclusion is twofold. First, if future research proved an account along these lines to be adequate from an empirical point of view, it would be a serious alternative to the current degree-based accounts developed within linguistics, due not only to its philosophical footing, but also to its advantages from the point of view of formal semantics. Second, the fuzzy approach to vagueness (as we know it today) needs to be refined and expanded, as the evidence provided by linguists clearly shows.

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Harmony in the Sequent Calculus: The Classical Case

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We aim to analyze one of the central notion of inferentialist semantics, the so-called *harmony* condition, from a sequent calculus perspective and in the case of classical logic.

According to Dummett (1991), harmony means that there is a balance between the conditions for the (correct) assertion of a certain complex sentence, and what can be drawn from this assertion. More precisely, the harmony principle is usually captured, in natural deduction, by what the so-called Prawitz's *inversion principle*: "by an application of an elimination rule one essentially only restores what had been established if the major premiss of the application was inferred by an application of an introduction rule" (Prawitz, 1965, p. 33). In other words, the rules for a connective c are harmonious in natural deduction when a c-detour (an introduction of c followed immediately by its elimination) can be "canceled out" in one reduction step.

First, we remark that by adopting a suitable translation from intuitionistic natural deduction to sequent calculus (like the one presented by von Plato (2003, §§ 2,3) and von Plato (2011)), it is possible to show that a c-detour corresponds to a cut where the cut-formula in both premises is principal (i.e. it comes from a rule for the connective c). For instance, in the case of conjunction:

$$
\begin{array}{c}
\dfrac{\dfrac{\begin{array}{c}\Gamma\\ \vdots\\ A\end{array} \quad \begin{array}{c}\Delta\\ \vdots\\ B\end{array}}{A\wedge B}\,\wedge_I}{\begin{array}{c}A\\ \vdots\\ C\end{array}}\,\wedge_{E_1}
\end{array}
\quad \text{corresponds to} \quad
\dfrac{\dfrac{\dfrac{\begin{array}{c}\vdots\\ \Gamma\vdash A\end{array} \quad \begin{array}{c}\vdots\\ \Delta\vdash B\end{array}}{\Gamma,\Delta\vdash A\wedge B}\,\wedge_R \quad \dfrac{\begin{array}{c}\vdots\\ \Gamma,\Delta,A\vdash C\end{array}}{\Gamma,\Delta,A\wedge B\vdash C}\,\wedge_{L_1}}{\Gamma,\Gamma,\Delta,\Delta\vdash C}\,\text{Cut}}{\overline{\Gamma,\Delta\vdash C}}\,\text{Ctr}_L
$$

So, a detour corresponds to what is usually called a *key-case* in the cut elimination procedure, and the harmony condition becomes the possibility of reducing the key-cases. Harmony is thus just a special case of cut elimination.

Second, we recall that usually (at least according to the Dummettian tradition) the inferentialist semantics provides a justification for intuitionistic inference rules, but not for classical ones. One of the most debated questions within inferentialist semantics concerns the possibility of justifying classical logic. More precisely, the question is whether the inference rules characterizing classical reasoning satisfy the harmony condition or not. We focus here on the rule of *reductio ad absurdum* (RAA), which is added to intuitionistic natural deduction to obtain classical natural deduction:

$$
\dfrac{\begin{array}{c}\Gamma,[\neg A]^{n}\\ \vdots\\ \bot\end{array}}{A}\,\text{RAA}\,[n] \tag{1}
$$

It is well known that RAA can be associated with some proof-transformation procedures (see Guerrieri & Naibo (2019) for a survey). What is under question is whether these proof-transformations can be seen as a *classical* harmony condition, analogous to the reduction steps for intuitionistic detours.

Consider for instance the case of what is called in Guerrieri & Naibo (2019) a *classical detour à la Prawitz*, that is, a configuration (in classical natural deduction) of the form

$$
\begin{prooftree}
\AxiomC{$\Gamma, [\neg(A\wedge B)]^{1}$}
\noLine\UnaryInfC{$\vdots$}
\noLine\UnaryInfC{$\bot$}
\RightLabel{RAA [1]}\UnaryInfC{$A\wedge B$}
\RightLabel{$\wedge_{E_1}$}\UnaryInfC{A}
\noLine\UnaryInfC{$\vdots$}
\noLine\UnaryInfC{C}
\end{prooftree}
\quad \text{which can be transformed into} \quad
\begin{prooftree}
\AxiomC{Γ}
\AxiomC{$[\neg A]^{2}$}
\AxiomC{$[A\wedge B]^{1}$}
\RightLabel{$\wedge_{E_1}$}\UnaryInfC{A}
\RightLabel{$\neg_E$}\BinaryInfC{$\bot$}
\RightLabel{$\neg_I$ [1]}\UnaryInfC{$\neg(A\wedge B)$}
\noLine\BinaryInfC{$\vdots$}
\noLine\UnaryInfC{$\bot$}
\RightLabel{RAA [2]}\UnaryInfC{A}
\noLine\UnaryInfC{$\vdots$}
\noLine\UnaryInfC{C}
\end{prooftree}
\tag{2}
$$

We claim that this kind of transformation can lead to the creation of a *key-case* in sequent calculus, which we know how to (cut-)eliminate. It is in this sense that we argue that classical reasoning can be considered as *harmonious*. However, in order to generate this key-case, another operation (different from cut elimination) is also needed.

Our idea is to translate the rule RAA depicted in (1) into the sequent calculus as follows:

$$
\begin{prooftree}
\AxiomC{$\Gamma, \neg A \vdash \bot$}
\RightLabel{$\neg_R$}\UnaryInfC{$\Gamma \vdash \neg\neg A$}
\AxiomC{}
\RightLabel{ax}\UnaryInfC{$A \vdash A$}
\RightLabel{$\bot_R$}\UnaryInfC{$A \vdash A, \bot$}
\RightLabel{$\neg_R$}\UnaryInfC{$\vdash A, \neg A$}
\AxiomC{}
\RightLabel{$\bot_L$}\UnaryInfC{$\bot \vdash$}
\RightLabel{$\neg_L$}\BinaryInfC{$\neg\neg A \vdash A$}
\RightLabel{Cut}\BinaryInfC{$\Gamma \vdash A$}
\end{prooftree}
$$

In this way, the classical detour *à la* Prawitz considered in the left-hand side of (2) becomes (when translated into the sequent calculus):

$$
\begin{prooftree}
\AxiomC{$\vdots$}
\noLine\UnaryInfC{$\Gamma, \neg(A\wedge B) \vdash \bot$}
\RightLabel{$\neg_R$}\UnaryInfC{$\Gamma \vdash \neg\neg(A\wedge B)$}
\AxiomC{}
\RightLabel{ax}\UnaryInfC{$A\wedge B \vdash A\wedge B$}
\RightLabel{$\bot_R$}\UnaryInfC{$A\wedge B \vdash A\wedge B, \bot$}
\RightLabel{$\neg_R$}\UnaryInfC{$\vdash A\wedge B, \neg(A\wedge B)$}
\AxiomC{}
\RightLabel{$\bot_L$}\UnaryInfC{$\bot \vdash$}
\RightLabel{$\neg_L$}\BinaryInfC{$\neg\neg(A\wedge B) \vdash A\wedge B$}
\RightLabel{Cut_1}\BinaryInfC{$\Gamma \vdash A\wedge B$}
\AxiomC{$\vdots$}
\noLine\UnaryInfC{$\Gamma, A \vdash C$}
\RightLabel{$\wedge_{L_1}$}\UnaryInfC{$\Gamma, A\wedge B \vdash C$}
\RightLabel{Cut_2}\BinaryInfC{$\Gamma, \Gamma \vdash C$}
\doubleLine\RightLabel{Ctr_L}\UnaryInfC{$\Gamma \vdash C$}
\end{prooftree}
\tag{3}
$$

Notice here that our translation (3) into the sequent calculus of the rule RAA already creates a cut of the form of a key-case with cut formula $\neg\neg(A\wedge B)$ (Cut_1). However, the formula that "disappears" after the transformation of the classical detour *à la* Prawitz considered in (2) is $A\wedge B$ (which corresponds to the cut formula for Cut_2 in our translation (3) into the sequent calculus). We then follow Urban (2001) and consider the possibility of letting the Cut_2 to pass over the Cut_1, so to obtain:

$$
\dfrac{\dfrac{\begin{matrix}\vdots\\ \Gamma,\neg(A\wedge B)\vdash\bot\end{matrix}}{\Gamma\vdash\neg\neg(A\wedge B)}\,\neg_R
\qquad
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A\wedge B\vdash A\wedge B}\,\mathsf{ax}}{A\wedge B\vdash A\wedge B,\bot}\,\bot_R}{\vdash A\wedge B,\neg(A\wedge B)}\,\neg_R \qquad \dfrac{}{\bot\vdash}\,\bot_L}{\neg\neg(A\wedge B)\vdash A\wedge B}\,\neg_L
\qquad
\dfrac{\begin{matrix}\vdots\\ \Gamma,A\vdash C\end{matrix}}{\Gamma,A\wedge B\vdash C}\,\wedge_{L_1}}{\Gamma,\neg\neg(A\wedge B)\vdash C}\,\mathrm{Cut}_2}{\dfrac{\Gamma,\Gamma\vdash A\wedge B}{\Gamma\vdash C}\,\mathrm{Ctr}_L}\,\mathrm{Cut}_1
$$

We can now attack the analysis of Cut_2 operating on the cut formula $A\wedge B$. This cut does not have the form of a key-case for $A\wedge B$ according to the cut elimination procedure. In order to obtain a key-case, it is necessary to operate an *expansion* on the initial sequent $A\wedge B\vdash A\wedge B$, i.e. to replace it with the derivation:

$$
\dfrac{\dfrac{\dfrac{}{A\vdash A}\,\mathsf{ax}}{A\wedge B\vdash A}\,\wedge_{L_1}
\qquad
\dfrac{\dfrac{}{B\vdash B}\,\mathsf{ax}}{A\wedge B\vdash B}\,\wedge_{L_2}}{A\wedge B\vdash A\wedge B}\,\wedge_R
$$

In this way, by permuting Cut_2 upward, one finally reaches the key-case in which the premise of the cut comes from the $\wedge_R$ of the expanded derivation above and the $\wedge_{L_1}$ used to derive $\Gamma,A\wedge B\vdash C$. The idea then is that when one passes from *classical* natural deduction to sequent calculus, she has to systematically expand proofs, ideally to atomic axioms. Therefore, in order to see the rule RAA as harmonious, one has to consider not only the operation of cut elimination, but also the operation of expansion for axioms. Harmony for classical logic seems then to have a more global flavor than the same notion form the intuitionistic case.

We will show that our approach can be generalized to the case of disjunction, but the expansion has to be done using the multiplicative rules for disjunction in sequent calculus: for these rules there is no translation into single-conclusion natural deduction respecting the standard introduction/elimination format. This explains why Prawitz's classical reductions were not originally given for the disjunction case.

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Expressing Validity: Towards a Self-Sufficient Inferentialism

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Inferentialism is the view that the meaning of an expression is fixed by the role that the expression plays in valid inferences, where "valid" includes more than just logical validity (Brandom, 2008; Peregrin, 2014; Steinberger and Murzi, 2017). So, for inferentialists, the basic semantic notion is (wider than logical) *validity*, which I express by "$\vdash$."[1] This contrasts with representationalist theories, which take *truth* or *reference* as their basic semantic notions.

Let's call a theory of meaning "self-sufficient" just in case it offers a satisfying account of the meanings of the sentences with which it is stated.[2] So a self-sufficient inferentialism offers an account of the meanings of sentences in which "valid" or "$\vdash$" occurs. Moreover, since a self-sufficient inferentialist theory applies to statements of itself, it should allow for self-reference. Unfortunately, giving an account of an expression for validity in a language that allows for self-reference is notoriously difficult (Beall and Murzi, 2013).[3] This paper aims to contribute to the construction of a self-sufficient inferentialism.

Problems with Expressing Validity

It may seem that an object language validity predicate, *Val*, should obey the following rules (where $\overline{A}$ is a name of A, and similarly for sets):

$$\frac{}{A, Val(\overline{A},\overline{B}) \vdash B}\ \mathrm{VD} \qquad\qquad \frac{A \vdash B}{\vdash Val(\overline{A},\overline{B})}\ \mathrm{VP}$$

Unfortunately, if we allow contraction, cut, and self-reference, these rules yield triviality via the v-Curry Paradox (Beall and Murzi, 2013).

The non-transitive approach to the problem rejects cut and adopts the non-transitive logic ST (Ripley, 2013). However, Barrio, Rosenblatt and Tajer (2017) have argued that while this saves us from triviality, it yields implausible results. We can derive $Val(\overline{\top},\overline{\kappa}), Val(\overline{\kappa},\overline{A}) \vdash Val(\overline{\top},\overline{A})$, while $\top \vdash \kappa$ and $\kappa \vdash A$ but $\top \nvdash A$. Thus, the non-transitive theory incorrectly proves the validity of instances of cut, instances that fail by ST's own lights. This flies in the face of self-sufficiency.

In previous work, I have responded to Barrio et al. by offering a non-transitive system, which I call NG, with a validity predicate that is "faithful" in the following sense (Hlobil, 2018). A validity predicate is faithful just in case $Val(\overline{\Gamma_1},\overline{\Delta_1})$, ... $Val(\overline{\Gamma_n},\overline{\Delta_n}) \vdash Val(\overline{\Theta},\overline{\Lambda})$ is provable iff $\Theta \vdash \Lambda$ follows from $\Gamma_1 \vdash \Delta_1$, ... $\Gamma_n \vdash \Delta_n$ via a derivable metarule.

Unfortunately, there are two problems with NG. First, NG does not prove of any argument that it is invalid. Arguably, however, the inferentialist should be able to establish at least some invalidities. Second, VD fails in NG. More precisely, in NG, if $A \nvdash B$, then $A, Val(\overline{A},\overline{B}) \nvdash B$. Here I suggest a way to address the first problem, but I argue that we should reject VD.

[1] This is not meant to deny that inferentialists can and do offer philosophical accounts of validity (e.g. Restall, 2005; Ripley, 2013). They usually do so in terms of norms governing discourse or thought.

[2] This is, of course, a variation on Tarski's (1943) notion of a semantically closed language.

[3] Note that since the inferentialist's notion of validity is wider than logical validity, it won't do to say that there is no problem regarding logical validity (Field, 2017; Ketland, 2012).

Proving Invalidities

The system I am presenting here, which I call STV, adds a validity predicate to (propositional) ST. As in ST, if Γ and Δ are *Val*-free and truth-free, then $\Gamma \vdash_{STV} \Delta$ iff the inference is classically valid. Moreover, STV has the following attractive properties:

(i) The validity predicate of STV is faithful, i.e., $Val(\overline{\Gamma_1},\overline{\Delta_1})$, ... $Val(\overline{\Gamma_n},\overline{\Delta_n}) \vdash_{STV} Val(\overline{\Theta},\overline{\Lambda})$ is provable iff the corresponding metarule instance is derivable in STV.

(ii) The validity predicate of STV captures all the invalidities of ST and, hence, of classical logic. That is, if $\Gamma \nvdash_{ST} \Delta$, then $\vdash_{STV} \neg Val(\overline{\Gamma},\overline{\Delta})$.

"Under the hood," the STV calculus differs from NG in two important ways. First, we let $Val(\overline{\Gamma},\overline{\Delta}) \vdash$ be an axiom if $\Gamma \cup \Delta$ contains only atoms and $\Gamma \cap \Delta = \emptyset$. (And we restrict the other axioms to atomic sequents.) Second, STV includes a restricted, context-mixing cut-rule. The rule allows us to use cut if all sentences in the involved sequents are *Val*-sentences and there are no open assumptions (like NG, STV allows us to assume and discharge sequents). This cut-rule makes sense despite the non-transitive setting because it captures the transitivity of the reasoning in sequent proof-trees. The rule captures invalidities because it allows us to formulate (unsuccessful) root-first proof searches in the object language.

While STV captures all invalidities of ST, it doesn't capture all invalidities of itself. I argue that this is okay because there are paradoxical sequents on which STV should not take a stand. According to this view, some sentences about validity should be neither asserted nor denied, which is in line with the non-transitive, bilateralist approach to paradox (Ripley, 2013). On my view, however, this applies not only to sentences in the object language but also to sequents.

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Higher-Order Skolem's Paradoxes

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In this short article, we will introduce some analogous higher-order versions of Skolem's paradox and we will assess the generalizability of two solutions for Skolem's paradox: the course-book approach and the Bays' one. We will show that Bays' solution to Skolem's paradox, unlike the course-book solution, can be generalized to solve the higher-order paradoxes without any implication about the possibility or order of a language in which mathematical practice is formalized.

Let S be one of the well-known first-order axiomatizations of set theory (for instance ZFC). Skolem's paradox (hereafter SP) is a seeming conflict between the *Downward Skolem–Löwenheim theorem* about S, and *Cantor's Theorem* within S. Suppose that S has a model. Since the language of S in standard formulations is countable, by the *Downward Löwenheim–Skolem theorem*, S has a countable model, M. Now, by *Cantor's Theorem*, S proves that there is an uncountable set, hence there is an a in the universe of M such that a is uncountable; that is to say, a satisfies in M the open formula which defines uncountability in the language of S. Insofar as M is countable, there are only countably many o in the universe of M such that $o \in a$. It seems then that within M, a is countable. Therefore, a is countable from one perspective (within the model), uncountable from another (within the theory).

According to the course-book approach, SP provides an evidence for the deficiency and semantical inadequacy of first-order theories for formalizing mathematical practice around countability and uncountability. Actually, SP is not alone. Firs-order logic has shortcomings in formalizing many other concepts of ordinary mathematics, too; for example, finitude, well-ordering, well-foundedness, powerset, etc.[1]

Now let us introduce some forms of higher-order Skolem paradox (hereafter HOSP). Let L be a language containing the first-order language with identity. Consider the following definition and theorem, both reported by Shapiro (1991, 147-8):

> *Definition. (Löwenheim number)* The Löwenheim number for L is the smallest cardinal κ such that for every formula φ of L, if φ is satisfiable, then it has a model with the cardinality at most κ.
>
> *Theorem. (Generalized Löwenheim)* If the collection of formulas of L is a set, then L has a Löwenheim number and the smallest extendible cardinal is an upper bound of it.

Now, an nth-order Skolem's paradox can be formulated as follows.[2] Let Sn be an nth-order axiomatization of set theory which can prove that there are extendible cardinals. And let κ be

[1] The course-book approach is mentioned and suggested several times in familiar course-books of introductory mathematical logic, such as Mendelson (2015) and van Dalen (2013).

[2] Our formulation of HOSPs appeals to the notion of *Löwenheim number*. Similarly, one can introduce other HOSPs by means of *Hanf number*, *set-Löwenheim number* and *set-Hanf number*, their definitions can be found in Shapiro (1991, 148). Here, we just focus on *Löwenheim* number, but the strategy can be reapplied for other numbers straightforwardly.

its Löwenheim number. For a cardinal larger than κ, we have a proof for a sentence φ which says that there exists a set whose cardinality is larger than κ. By the *Generalized Löwenheim theorem*, this sentence has a model, M, with the cardinality of at most κ. M satisfies "there exists a set whose cardinality is larger than κ," hence there is a in the universe of M such that the size of a is larger than κ. While the cardinality of M is at most κ, there are at most κ objects o in the universe of M such that $o \in a$. It seems then that within M, a's size is at most κ. Therefore, a's size is at most κ from one perspective (within the model), larger than κ from another (within the theory).[3]

Reapplying the course-book approach to handle these nth-order paradoxes might seem to be appealing. Accordingly, the paradox could be solved by going to a higher-order language, but again an analogous higher-order paradox can be formulated for the higher-order language; and so on. Thus, the course-book approach to handle SP cannot be generalized to solve the parallel HOSPs, unless it is augmented with the claim that there is no unique language that the practice of mathematics (set theory, particularly) can be formalized within it. It might be so, but it seems more plausible if SP and its counterparts can be handled without conceding such radical claim about mathematical practice and its formalization. In defense of the course-book approach, one might suggest that these new paradoxes are not as philosophically valuable as the original SP, for the large cardinals are not as much involved in mathematical practice as concepts like (un)countability. Second-order logic, though not apt for the *whole* practice of mathematics, is adequate for its *ordinary* part. This is a non-starter, however. Mathematical practice is not something stable and closed-end. It is foreseeable that large cardinals will become more involved in mathematical practice than they are now. Furthermore, a valuable portion of the practice of set theory is already devoted to the study of large cardinals. Therefore, an alternative approach seems to be attractive.

Bays (2000) provides a solution for SP which appeals to an equivocation between model theoretic and plain English interpretations of "$\exists x$(x is uncountable)". The distinctive feature of Bays' solution is that it is silent with respect to the mathematical practice and its possible formalization. Particularly, unlike the course-book approach, SP is not resolved by moving to the second-order language; all that is said is done in a first-order language.

Now, we generalize Bays' solution for HOSPs. To do so, we will only consider the HOSP. Treating other HOSPs would be similar. Let κ be the Löwenheim number of a standard second-order axiomatization of set theory, S2. And let M be a model for S2 of cardinality at most κ. Its existence is ensured by the *Generalized Löwenheim Theorem*. Now consider $\Psi(x)$ to be an articulation of "x is of cardinality larger than κ" in the language of S2. Since M satisfies S2, there is an $m^* \in M$ such that $M \vDash \Omega[m^*/x]$. We can now formulate the second-order HOSP: (hereafter *argument (A^*)*)

1. M is a model of S2 of cardinality at most κ.

2. $\Psi(x)$ says that "x is of cardinality larger than κ".

3. $M \vDash \Psi[m^*/x]$.

$\therefore$ 4. $\{x \mid x \in m^*\}$ is of cardinality larger than κ.

5. If M is of a cardinality at most κ, so is $\{x \mid x \in m\}$.

$\therefore$ 6. $\{x \mid x \in m^*\}$ is of cardinality at most κ.

[3] Higher-order Skolem paradoxes is already mentioned by Hart (2000) based on considerations given by Hasenjaeger (1967).

Let $\Omega_E(x)$, $\Psi_E(x)$ be second-order ordinary English interpretations of $\Omega(x)$, and $\Psi(x)$, respectively. $\Psi_E(x)$ represents the lack of a one to one function from x into the smallest ordinal with the cardinality κ. Furthermore, like $\Omega_M(x)$, $\Psi_M(x)$ can be put to give a model-theoretic semantics to $\Psi(x)$.

Since the most natural justification of line 3 is to interpret $\Psi(x)$ as $\Psi_M(x)$ and the most natural justification of line 4 is to interpret $\Psi(x)$ as $\Psi_M(x)$, then the validity of argument (A*) will be dependent on the truth of the following conditional:

$$\forall m \in M[\Psi_M(m) \Rightarrow \Psi_E(\{x \mid x \in m\})].$$

It is left to argue that this conditional is false. Like Bays' original solution, the semantics of $\Psi_E(x)$ and $\Psi_M(x)$ might differ at least in two ways. First, the semantics of $\Psi_E(x)$ and those of $\Psi_M(x)$ may differ for atomic formulas, because the semantics of $\Psi_E(x)$ interpret the symbol "$\in$" as a simple membership. But, the semantics of $\Psi_M(x)$ interpret "$\in$" regarding the interpretation function for M. Second, there are other disparities in more complicated formulas. For second-order quantifiers, the semantics of $\Psi_E(x)$ interprets "$\exists x$" as "there is a set x, such that", whereas the semantics of $\Psi_M(x)$ interpret the expression "$\exists x$" corresponding to "there is a set $x \in M$, such that". These asymmetries between $\Psi_E(x)$ and $\Psi_M(x)$ guarantee that the conditional under consideration is not true.

We have seen how neglecting semantical disparities between $\Psi_E(x)$ and $\Psi_M(x)$ leads to the second-order paradox. So, as Bays did for SP we can conclude that the analogous second-order paradox neither have any possible consequence about the suitable order for the language by which we may formulate mathematical practice nor imply that there is not a unique language in which we can formulate the practice. Namely, unlike the course-book approach, the second-order paradox is not resolved by moving to a higher-order language for a second-order language. In sum, Higher-order Skolem's paradoxes, as introduced here, are puzzling as much as the original Skolem's paradox. The course-book approach to solve the first-order paradox, however, is not generalizable to solve the higher-order paradoxes, unless one concedes that there is no language in which the practice of mathematics (especially set theory) can be formalized. Fortunately, this is not the end of the story. Bays' solution to the original paradox has the power to be generalized to solve the higher-order paradoxes. Like Bays' original solution, the generalized one does not have any implication for the (im)possibility of a language in which the practice of mathematics may be formalized. This is a virtue for Bays' solution, and in effect for our generalization of it, that makes them preferable to the coursebook approach.

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Logical Approaches to Maximizing Expected Utility

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Maximizing expected utility is a central concept within decision theory and its multi-agent cousin, game theory. Reasoning about expected utilities encompasses a variety of game theory's core aspects including the agent's beliefs, preferences and their strategy space, pure or mixed. These frameworks presuppose quantitative notions in various ways, both on the input and output side. On the input side, maximization of expected utility requires a quantitative, probabilistic representation on the agent's *beliefs*. Besides, also the agents' preferences over the various pure *outcomes* need to be given quantitatively rather than by a mere qualitative order. Finally, the scope of maximizing is sometimes taken to include mixed strategies, requiring a quantitative account of mixed actions.

Besides the input, also the output of utility maximizing reasoning is quantitative: Expected utilities are often understood as a quantitative preference, or evaluation of available actions, again pure or mixed.

In this talk I present first steps towards a logical framework for maximizing expected utilities. The current approach builds on two core subfields of logic: qualitative frameworks for games and for probabilities. Towards a start we introduce these in turn.

Logical models of games exist for both, strategic and extensive form games, see (van Benthem and Klein, 2019) for an overview. For the sake of brevity, we restrict this exposition to two player strategic form games. Notably, this restriction is not substantial, as extensive form games work in a similar way. To start describing the logic with its components, we once and for all need to fix a set of atomic propositions $\mathfrak{A}$. We also fix finite sets M_1 and M_2 of possible moves available to player 1 and 2. Before diving into the exact logical framework, we define the **semantics** with the underlying two player extensive form game models.

Definition An **extended game model** $(M_1 \times M_2, V, p_1, p_2)$ consists of a set of worlds $W := M_1 \times M_2$, a valuation $V : W \to 2^{\mathfrak{A}}$ and functions p_1, p_2 such that $p_1 : M_2 \to [0,1]$ and $p_2 : M_1 \to [0,1]$ are probability functions. We moreover define equivalence relations $\sim_1$ and $\sim_2$ by $(m,n) \sim_1 (m',n')$ iff $m = m'$ and $(m,n) \sim_2 (m',n')$ iff $n = n'$.

The interpretation is that each possible choice of player i picks out an $\sim_i$-equivalence cell of possible worlds this agent cannot distinguish. By construction, each combination of choices, one for each player, fixes a unique outcome cell. The functions p_1 and p_2 structure the player's uncertainty: While players do not know or learn about their opponent's choices, they may entertain non-conclusive beliefs on which choices will be made. These beliefs are recorded by the functions p_1 and p_2. Finally, we should emphasize that extended game models are highly abstract in that they do not incorporate any utility function. Rather, extended game models record properties of outcome worlds through the valuation function V. Players utilities, we assume, will then be derived from the outcome's properties in a way that is not explicitly modeled here. Extended game models as presented here hence correspond to the game forms of (van Benthem, 2014)

On the **syntactic** side, the logic incorporates a weak probabilistic framework that allows players to express their beliefs about the game's outcomes. These are recorded by diadic probabilistic operators $p_i(\varphi, m)$ for $i = 1,2$ with φ a formula in the propositional language over $\mathfrak{A}$, and $m \in M_i$. These operators express the probability of φ after making move m.

Formally, for defining the language a set of probabilistic terms is defined as

$$\Phi = p(\varphi|m)|\Phi + \Phi$$

with φ in $\mathscr{L}_0$ the propositional language over $\mathfrak{A}$ and $m \in M_1 \cup M_2$. The language $\mathscr{L}$ is then defined with the following BNF, combining a classic probabilistic logic (Delgrande and Renne, 2015) with a standard strategic language.

$$\Psi = \Phi \geq \Phi, \neg\Psi, \Psi \wedge \Psi$$

.

With this in hand, we can move towards expected utitlity reasoning. First assume a perspective motivated by Harrenstein *et al.* (2001), where each agent pursued a goal that is formulated in $\mathscr{L}_0$, the propositional language over $\mathfrak{A}$. We assume the agent receives high utility, say 1, if she reaches her goal and 0 else. In this case, move M dominates move n of player 1 iff $p_1(\varphi|m) \geq p_1(\varphi|n)$ where φ is player 1's goal. The fact that some m maximizes i's expected utility among her pure strategies is hence expressed by

$$\text{max-EU}_i(m) := \bigwedge_{n \in M_i} p_1(\varphi|m) \geq p_1(\varphi|n),$$

exploiting that M_1 is finite.

Next, we generalize this case. Assume that players pursue finitely many goals $\varphi_1^i, \ldots, \varphi_n^i$ and assign rational utility values to each. Then we show that there exist probabilistic terms Φ_1, Φ_2 such that m dominates n in terms of expected utility for player i iff $\Phi_i^m \geq \Phi_i^n$, where Φ_i^k denotes a substitution instance of Φ_i where all occurrences of move-variables (so all second components of atomic terms $p(\varphi|o)$) are replaced by k. Again, the fact that n maximizes expected utility among pure strategies can be expressed as Max-EU$_i(m) := \bigwedge_{n \in M_i} \Phi_i^m \geq \Phi_i^n$. In particular, this logic is sufficiently rich to express utility maximization among pure strategies within finite games, provided that players assign rational-valued utilities.

Lastly, we illustrate a further application of this logic. For this, we slightly re-interpret the models give above by assuming that probability functions do not track player uncertainy, but instead mixings between different strategies. That is, player i is no longer certain which strategy she plays. Instead, she mixes with probabilities known to her. We assume these mixing parameters to be common knowledge among all players involved. Moreover, we enrich the language to contains atoms $\mathfrak{m}$, one for each $m \in M_1 \cup M_2$, denoting that the corresponding player has played move m. In this case, recall that a mixed strategy of player 1 where she plays move m_k with probability p_k maximizes expected utility, given the opponent's current move n iff $EU(m_k, n) \geq EU(m_l, n)$ for any $m_k, m_l \in M_i$ with $p_k > 0$. Notably, the latter condition is finitely expressible within our language. Indeed, given the usual assumptions of games being finite and utilities rational-valued, we will show that there are finite formulas $\mathsf{mix} - \mathsf{EU}_i$ for $i = 1, 2$, expressing that the current strategy mixing maximizes expected utility in light of the opponents mixed strategy. In particular, this implies that Nash equilibria become expressible within our logic, as a game is in Nash equilibrium if $\mathsf{mix} - \mathsf{EU}_1 \wedge \mathsf{mix} - \mathsf{EU}_2$.

To end the presentation, we provide a complete axiomatization of the logic of extended game models.

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Ability and Knowledge

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Imagine that I place all the cards from a deck face down on a table and ask you to turn over the Queen of Hearts. Are you able to do that? In a certain sense, yes – this is referred to as causal ability. Since you are able to pick any of the face-down cards, there are 52 actions available to you, and one of these guarantees that you turn over the Queen of Hearts. However, you do not know which of those 52 actions actually guarantees the result. Therefore, you are not able to turn over the Queen of Hearts in an epistemic sense. I explore this epistemic qualification of ability and three ways of modelling it – namely, epistemic transition systems, standard epistemic stit models, and labelled stit models.

First, Naumov and Tao (2018) present epistemic transition systems as labelled directed graphs – with vertices denoting states and labelled directed edges denoting transitions from one state to another – supplemented by an indistinguishability relation on vertices to capture knowledge. As such they could be thought of as Kripke models of modal logic S5 with knowledge, to which transitions controlled by the agents' actions are added. Labels on the edges represent the choices that the agents make during the transition. Naumov and Tao use this framework to study three modalities – epistemic modality K, strategic modality S, and know-how modality H. In particular, a formula of the form $\mathsf{H}_i\varphi$ is to be read as saying that agent i knows how to (reach) φ. More precisely, Naumov and Tao stipulate that an agent knows how to φ if and only if there is a strategy available to her that she knows guarantees it. Phrased differently, if and only if there is a strategy that achieves φ in all the states that the agent cannot distinguish from the current state.

Second, stit theory grew out of a modal tradition in the logic of action. It originates from the series of papers by Belnap, Perloff, and Xu, culminating in their book (Belnap et al., 2001). In this tradition, the agency of an individual is characterised by a modal operator of the form $[i\ \mathsf{stit}]\varphi$, which is to be read as saying that agent i sees to it that φ holds. In particular, stit semantics is cast against the background of a theory of indeterministic time, where the world is represented as moments ordered in a tree of histories, resulting in a branching-time structure (Belnap et al., 2001; Horty, 2001). Briefly, a stit model is a tuple $\langle M, H, <, Ags, (Act_i^m), \pi\rangle$, where M is a set of moments, $H \subseteq 2^M$ is a set of histories, $<$ is a strict partial ordering on M without backward branching, Ags is a finite set of agents, for each moment m and each agent i it holds that $Act_i^m \subset 2^{H_m}$ is a finite set of actions available to agent i at moment m (constituting a partition of $H_m = \{h \in H|\ m \in h\}$ – the set of histories passing through moment m), and $\pi : P \to 2^{M\times H}$ is a valuation. In stit theory, actions mean "action tokens – particular, concrete actions, each occurring at a single point in space and time" (Horty and Pacuit, 2017, p. 617). Applying the standard modal treatment of knowledge, an epistemic stit model is a stit model supplemented with a set of indistinguishability relations $\sim_i$, one for each agent $i \in Ags$ (Xu, 2015).

Third, labelled stit models, introduced by Horty and Pacuit (2017), are epistemic stit models extended with action types – each action token is assigned a label indicating the type it instantiates. Horty and Pacuit argue for the need of an explicit treatment of action types by claiming that "if the epistemic sense of ability requires that some single action must be known by agent i to guarantee the truth of φ, then this must be the action type, not one of its various tokens" (Horty and Pacuit, 2017, p. 626 – notation adapted). In order to capture the epistemic sense of agency they introduce a new epistemic modality of the form $[i\ \mathsf{kstit}]\varphi$, which is to be read as saying that agent i sees to it that φ holds, in an epistemic sense. Horty and Pacuit then characterise epistemic ability using a combination of epistemic agency and impersonal possibility in the form $\Diamond[i\ \mathsf{kstit}]\varphi$.

I show that both the analyses of knowing how in epistemic transition systems and of epistemic ability in labelled stit models can be simulated using a combination of impersonal possibility, knowledge and agency in standard epistemic stit models. What is more, the standard analysis of the epistemic qualification of ability relies on action types – as opposed to action tokens – and states that an agent has the epistemic ability to do something if and only if there is an action type available to her that she knows guarantees it. I argue, however, that these action types are dispensable. This is supported by the fact that both epistemic transition systems and labelled stit models rely on action types, yet their associated standard epistemic stit models do not. Instead, these models make use of the notion of knowingly doing which has been introduced and studied by Broersen in a series of papers (Broersen, 2008, 2011a,b). He writes that "'knowingly doing' is an epistemic qualification concerning an action" (Broersen, 2011a, p. 144) and expresses it by a simple combination of knowledge and agency – $\mathsf{K}_i[i\ \mathsf{stit}]\varphi$. Adding impersonal possibility yields the characterisation of epistemic ability in the form $\Diamond\mathsf{K}_i[i\ \mathsf{stit}]\varphi$, which is to be read as saying that it is possible that agent i knowingly sees to it that φ holds.

Formally, a general correspondence result is achieved by a systematic transformation of an epistemic transition system to a labelled stit model and of a labelled stit model to an epistemic stit model. First I show that the analysis of knowing how in a given epistemic transition system corresponds to the analysis of epistemic ability in the transform labelled stit model, and then that the analysis of epistemic agency in a given labelled stit model corresponds to the analysis of knowingly doing in the transform epistemic stit model. This means that the analyses of knowing how by Naumov and Tao (2018) and of epistemic ability by Horty and Pacuit (2017) can both be simulated in standard epistemic stit theory without involving action types.

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Reasoning in Four Valued Probabilities

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Belnap and Dunn introduced a four valued propositional logic which was designed to deal with incomplete or contradictory information (see e.g. Belnap (1992)). It extends the classical approach in that the propositions can not simply be true or false, but also both or neither. Formally there are two approaches to capture this phenomena: either to maintain classical truth values T,F and to define evaluation as a set function into $\mathscr{P}(\{T,F\})$ or to introduce four explicit truth values True, False, Neither, Both. The Belnap-Dunn four-valued logic, pursuing the first of these attempts, has been extensively studied since its introduction. It has proved fruitful in the study of rational agency and the rational agents attitude towards the truth or falsity of propositions in more realistic contexts. Recently, two major attempts have been undertaken to identify probabilistic counterparts of Belnap-Dunn logic.

A first attempt to generalize the Belnap-Dunn logic probabilistically has been undertaken by Dunn (2010) and Childers et al. (2019). Michael Dunn's four valued probability is a function which assigns to each event A a four valued vector (*belief, disbelief, uncertainty, conflict*) of non-negative numbers. The vector is normalized, i.e. its entries add up to one. No further dependencies between the four entries are assumed.

(Childers et al., 2019) build their approach on the insight, that the process of confirmation of a hypothesis in scientific practice might be different from the process of its refutation. So they assume the probabilities of a proposition and its negation are independent. This approach is similar to double valuation approach in logic, where the truth of a proposition is independent of its falsity. It comprises three axioms (cf. also Priest (2006) or Mares (2006)):

1. $0 \leq p(\varphi) \leq 1$
2. $\varphi \vDash_L \psi$ then $p(\varphi) \leq p(\psi)$
3. $p(\varphi \wedge \psi) + p(\varphi \vee \psi) = p(\varphi) + p(\psi)$

As these axioms are weaker than Kolmogorov's, (the additivity axiom is replaced by inclusion /exclusion) they give rise to a non-standard notion of probability. In particular φ and $\neg\varphi$ are not complementary in the usual sense. Instead, they are connected by a weaker condition:

$$p(\varphi \wedge \neg\varphi) + p(\varphi \vee \neg\varphi) = p(\varphi) + p(\neg\varphi),$$

which allows for $p(\varphi \wedge \neg\varphi) > 0$ (positive probability of gluts) and $1 - p(\varphi \vee \neg\varphi) > 0$ (positive probability of gaps).

Both approaches are intertranslatable . The relation of "probability" (of φ) in the latter approach to "belief" in Dunn's setting, (i.e. b in the vector (b,d,u,c)) is the same as the relation of "at least true" to "exactly true" in the standard relevant logic, where, A is "exactly true" when $T \in v(A)$ and $F \notin v(A)$. Similarly, Dunn's b is a "pure" belief in A , and can hence be expressed as

$$b(A) = p(A) - p(A \wedge \neg A)$$

where p stands for the second approach's probability function. Similar translations apply to disbelief and the remaining components:

$$d(A) = p(\neg A) - p(A \wedge \neg A) \quad c(A) = p(A \wedge \neg A) \quad u(A) = 1 - p(A \vee \neg A)$$

It is easy to check that the b, d, u and c hence defined sum up to unity. Dunn's approach, however, is more expressive than the Prague setting. In translating from the double valuation approach, at least one of the conflict or uncertainty component has value 0 after the translation. Lastly, we remark that this translation also works also in the other direction. Defining p as

$$\begin{array}{ll} p(A) = b_A + c_A & p(\neg A) = d_A + c_A \\ p(A \vee \neg A) = 1 - u_A & p(A \wedge \neg A) = c_A \end{array}$$

Besides clarifying the relation between the two approaches introduced above, this contribution expands either non-standard probability framework by considering logical relationships between different formulas. Being informed about the probabilities of φ and ψ, we may for instance ask about the (non-standard) probability of $\varphi \wedge \psi$. Relatedly, we may be interested in what happens if the agent learns ψ, i.e. we may ask about $p(\varphi|\psi)$. Lastly, we may inquire into combining probabilites of different sources. That is, if two agents differ in their non-standard probabilities of φ, we can ask about ways for combining these into a joint belief.

Notably, the frameworks discussed have remained largely silent about these question. Dunn's Dunn (2010), for instance, suggested a definition for the non-standard probability $\varphi \wedge \psi$. This definition assumed φ and ψ to be probabilistically independent, irrespective of their exact form or content. This raises various problems. For instance, $p(\varphi \wedge \varphi)$ need not be the same as $p(\varphi)$, as the former treats both occurrences of φ in the former formula as probabilistically independent from each other.

In the current contribution, we suggest an alternative, semantically based approach to the probability of conjunctions $p(\varphi \wedge \psi)$. In a generalization of Bayes' rule, we moreover show how such conjunctive belief relates to conditional belief. As it turns out, defining conditional beliefs requires to disambiguate various possible readings of the event learned. Within classic probabilistic reasoning, learning ψ and forming $p(\varphi|\psi)$ is a shorthand for learning that ψ is *true*. Learning that ψ is false, similarly, relates to $p(\varphi|\neg\psi)$. In a generalized setting, we could learn more than the truth or falsity of ψ: We could, for instance, learn that it is at least true, at least false, exactly true, true and false.... We define a notion of conditional belief for each of these possible updating events. Moreover, we also explore various approaches to settling disagreement between non-standard probabilistic beliefs, i.e. for merging two non-standard probability assignments on φ.

Lastly, we relate our discussion to the underlying logical spaces. As is well known, every classical probability assignment over a finite propositional language can be translated into a probability assignment over the set of valuations of that language's atoms. We explore to generalizations of this fact to non-standard probabilities. In the first case, non standard probability assignments concern to assign two values, truth and falsity, to the valuations of a classical propositional logic. In the second case, non standard probabilities are given by truth assignments on the Lindenbaum algebra of a particular non-classical logic.

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Revision Operator Semantics

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There are many logics containing a conditional connective $>$, including classical logic with material implication, modal logic with strict implication, normal and classical conditional logics with variably strict conditionals, the conditional logic of dynamic semantics, intuitionistic logic, relevance logic, and the new hyperintensional logic Hype. Whereas some relations between these are known, others remain unexplained. What is the relation between relevance logic and classical conditional logic, or between logics in dynamic semantics and the latter two? This paper builds a bridge, by introducing a semantics where each logical operator is defined based on a different revision operator, plus an additional relation. The above logics are shown to arise from special cases, by tuning the semantic parameters.

Consider a language $\mathscr{L}$ over variables Var, with $\neg$, $\wedge$ and a conditional $>$.

Definition 1. $\mathfrak{M} = \langle S, R^{\neg}, R^{\wedge}, R^{>}, L, M, E, V \rangle$ is a **revision model** for $\mathscr{L}$ (with $\neg, \wedge, >$) iff

1. $S \neq \emptyset$, the **states**.
2. $V : S \longrightarrow \wp(\text{Var})$ is a total function, the **valuation**.
3. $R^{\neg} : S \times \mathscr{L} \longrightarrow \wp(S)$ is a partial function, the **$\neg$-revision**.
4. $R^{\wedge} : S \times \mathscr{L} \longrightarrow \wp(S)$ is a partial function, the **$\wedge$-revision**.
5. $R^{>} : S \times \mathscr{L} \longrightarrow \wp(S)$ is a partial function, the **$>$-revision**.
6. $L \subseteq S \times \mathscr{L}$, the **lacking operator**.
7. $E \subseteq S \times \mathscr{L}$, the **endorsing operator**.
8. $M \subseteq S \times S$, the **matching operator**.

The R^x (for $x \in \{\neg, \wedge, >\}$) are revision operators. Given a state $s \in S$ and a sentence $\varphi \in \mathscr{L}$, $R^x(s, \varphi)$ selects admissible φ-revisions of s according to the mode x. Sometimes it will be usefull to think of $R^x(., \varphi)$ as a relation, then I write $sR^x_\varphi s'$ for $s' \in R^x(s, \varphi)$ and $R^x_\varphi(s) = \{s' : sR^x_\varphi(s)\}$. $sL\varphi$ means that state s lacks φ. $sE\varphi$ means that s endorses φ and $s'Ms$ means that s' matches s. The standard interpretation of lacking is non-satisfaction (L is $\nvDash$). Another interpretation is that φ is disbelieved or that a φ-lacking state is φ-absurd, or simply absurd or 'non-normal'. The standard interpretation of endorsement is satisfaction $\vDash$, but for some purposes it will be interpreted as belief. The standard interpretation of matching is that the relation M contains the identity relation – at least identical states match. Another interpretation of $s'Ms$ is that s' resembles s in some respects. This becomes clearer once we consider the truth clauses and examples:

Definition 2. **Satisfaction** in a state in a revision model is defined inductively as follows:

1. $s \vDash p$ iff $p \in V(s)$.
2. $s \vDash \neg\varphi$ iff for all $s' \in R^{\neg}(s, \varphi)$, $s'L\varphi$.

3. $s \vDash \varphi \wedge \psi$ iff there are s', s'' such that $s' \in R^{\wedge}(s,\varphi), s'' \in R^{\wedge}(s',\psi)$ and $s''Ms$.

4. $s \vDash \varphi > \psi$ iff for all $s' \in R(s,\varphi)$, $s'E\psi$.

We use 'satisfaction' instead of truth to allow for a flexible interpretation: $\vDash$ can mean acceptance, belief, knowledge, or truth.

The classical semantic truth definitions for $\neg, \wedge$ and $>$ (seen as material implication) can be obtained from the above mentioned standard interpretations augmented by conditions on R^x. Negation is semantically classical if $R^{\neg}$ is the identity and lacking L is $\nvDash$. Because then we have

- $s \vDash \neg\varphi$ iff $\forall s'$ if $s' = s$ then $s' \nvDash \varphi$ iff $s \nvDash \varphi$.

Conjunction is classical if $s' \in R^{\wedge}(s,\varphi)$ is "$s = s' \vDash \varphi$" and M is the identity. Because then

- $s \vDash \varphi \wedge \psi$ iff there are s', s'' such that $s = s' \vDash \varphi$, $s' = s'' \vDash \psi$ and $s'' = s$ iff $s \vDash \varphi$ and $s \vDash \psi$.

The conditional is the material implication provided $R^{>}(s,\varphi) = \{s\}$ if $s \vDash \varphi$ and else empty, and E is $\vDash$. Because then

- $s \vDash \varphi > \psi$ iff if $s \vDash \varphi$ then for all $s' = s$, $s' \vDash \psi$, or $s \nvDash \varphi$ iff $s \vDash \varphi$ implies $s \vDash \psi$.

When an operator is classical, we can thus drop the defining semantic objects from the model. Conversely, if they don't appear, we assume them to be classical by convention.

The remarks that follow are based on showing point equivalences. That is, when I say "we obtain logic L if we restrict the revision models by imposing constraints X on the operators", I really mean that the model class M satisfying the constraint is such that every model in that class is point equivalent to a model in some class M' and vice versa, where it is known that M' has the sound and complete logic L. In what follows, these point equivalences are sketched by examining the inductive step for the relevant operator (assuming the induction hypothesis).

With this phrasing and the above remark on classicality, we obtain classical logic (without $\vee$ and where $>$ becomes the material implication) if we take $\langle S, V \rangle$ with S all atomic truth functions $s : \mathrm{Var} \longrightarrow \{0,1\}$, $V(s) = \{p : s(p) = 1\}$ (and $\neg, \wedge, >$ are assumed classical by our convention). We obtain the strict implication $\Box(. \rightarrow .)$ (where $\varphi \rightarrow \psi$ abbreviates $\neg(\varphi \wedge \neg\psi)$), as in a Kripke model, if we take $\langle S, R^{>}, V \rangle$ where $R^{>}_{\top} \subseteq S^2$ and $R^{>}_{\varphi}(s) = R^{>}_{\top}(s) \cap [\varphi]$ (and $\neg, \wedge$ are classical). To see this, one considers the Kripke relation $R = R^{>}_{\top}$. Then we obtain

- $s \vDash \varphi > \psi$ iff $\forall s' \in R(s) \cap [\varphi]$, $s' \vDash \psi$, iff $s \vDash \Box(\varphi \rightarrow \psi)$.

Normal and non-normal modal logics can also be obtained, as well as intuitionist logic.

If we consider $\langle S, R^{>}, E, V \rangle$, where each $R^{>}(s,\varphi) = \{s_{\varphi}\}$ (and $\neg, \wedge$ classical), $R^{>}$ together with E give rise to $F(s,\varphi) := \{\psi : s_{\varphi}E\psi\}$. This induces a sentence selection model, where the conditional is based on a sentence selection $F : W \times \mathscr{L} \longrightarrow \wp(\mathscr{L})$ and is by $w \vDash \varphi > \psi$ iff $\psi \in F(w,\varphi)$. Because

- $s \vDash \varphi > \psi$ iff $s_{\varphi}E\psi$ iff $\psi \in F(s,\varphi)$.

Chellas' (1975) minimal frames for classical conditional CE result when requiring E and $R^{>}$ to be propositional (i.e., they do not distinguish equivalent sentences). Normal conditional logics CK results if we ask that $E(s) = \{[\psi] : sE\psi\}$ is a filter and for Stalnakers' conditional logic it needs to be an ultrafilter. Thus we can model all existing variably strict conditionals.

The following examples are more complicated and sketching them in more detail does not make much sense in this abstract. We obtain standard dynamic semantics (Veltman (1996), Gillies (2004)), if we take $S = \wp(W)$, $V(s) = \{p : s \subseteq v(p)\}$, $s' \in R^x(s,\varphi)$ iff $s[\varphi] = s'$ for all $x \in \{\neg, \wedge, >\}$, where $s[\varphi]$ is recursively defined as in the dynamic semantic, $s'L\varphi$ iff $s' = \emptyset$, M is $=$ and E is $\vDash$. Here, only $\wedge$ is classical. We obtain models used by Urquhart (1972) for relevance logic (in $\mathscr{L}$ without $\neg$) when considering $\langle S, R^{>}, V\rangle$, where E is $\vDash$, V is 'atomically hereditary', $R^{\geq}_{\top}$ is a partial order and $R^{>}_{\varphi}(s) = R^{\geq}_{\top}(s) \cap [\varphi]$. Other relevance logics extending the weak implication calculus $\mathsf{R}_{>}$ can be simulated. Leitgeb's (2018) new Hype logic is a special case, by reinterpreting conditions on the fusion $\circ$ as conditions on the least upper bound of states with respect to partial order $\leq$ corresponding to $\circ$. For this we interpret $R^{>}$ as a partial order $\leq$ plus satisfaction. Additionally, we simulate the negation of Leitgeb's incompatibility relation by $R^{\neg}_{\varphi}$ to obtain Hypes' negation.

In all above cases, M is the identity and $\wedge$ turns out classical. In general, $\wedge$ is a hidden modal operator. To see this, suppose that $M = S^2$ and $R^{\wedge}_{\varphi}$ is a relation R augmented by φ-satisfaction. Then $\varphi \wedge \psi$ is of the form $\Diamond(\varphi \wedge_c \Diamond \psi)$, where $\wedge_c$ is classical conjunction and $\Diamond$ is a Kripke possibility based on R. For $s'' \in M(s)$ iff $s''Ms$, the intersection $M(s) \cap R(s')$ acts as a new relation, so that conjunction is of the form: $\Diamond_1(\varphi \wedge_c \Diamond_2 \psi)$, where $\Diamond_2 \psi$ implies $\Diamond_1 \psi$. If we impose no condition on $R^{\wedge}$ and take $s''Ms$ to mean $s'' \vDash \top$, conjunction is of the form $\varphi \Diamond\!\!\rightarrow (\psi \Diamond\!\!\rightarrow \top)$, where $\Diamond\!\!\rightarrow$ is the dual of a variably strict conditional $\Box\!\!\rightarrow$. Thus $\varphi \wedge \psi$ means (using duality) "it is not the case that if φ, ψ is impossible (according to the outer modality of $\Box\!\!\rightarrow$)". If import-export holds for $\Box\!\!\rightarrow$, we obtain "$\varphi \wedge_c \psi$ is possible". Since for all known logics above, M is the identity, we could as well have chosen then universally quantified version of the truth condition for $\wedge$. We would obtain dual expressions for all modals (conditionals) occuring in the versions using non-identity of M. Another option is to generalise the clause from update semantics ($s \vDash \varphi \wedge \psi$ iff $s[\varphi][\psi] = s$). Suppose $R_{\varphi}(s) = \{s[\varphi]\}$ and conceive of M as an equivalence (or similarity) relation $\sim$, then conjunction has the dynamic form:

- $s \vDash \varphi \wedge \psi$ iff $s[\varphi][\psi] \sim s$.

Intuitively, s satisfies $\varphi \wedge \psi$ if updating first by φ then by ψ yields a state which remains similar to s in relevant respects.

All these results can also be used to compare the different logics as triggered by different semantic implementations of the Ramsey test (Ramsey, 1929, 1990).

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Sortal Difference and Aristotelian Logic

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The general idea is the reconstruction of Aristotelian logic (AL) within the framework of so called sortal logic (SL). More concretely, I will focus on the concept of difference. The concept of difference plays very important role in AL, especially in the theory of categories. On the other hand, in SL this concept does not play major role. But, the topic of identity is relatively often discussed in SL. My main idea is to define sortal difference in SL a then try to use it as an explanation and formal counterpart of Aristotelian concept of difference. As a consequence, the notion of internal negation can be clarified, both in AL and SL.

The classical notion of identity is usually defined with the help of Leibniz Law, i.e. $\forall x \forall y(x = y \equiv \forall P(Px \equiv Py))$. Now, the notion of difference can be analogically stated as $\forall x \forall y(x \neq y \equiv \exists P(Px \wedge \neg Py))$.

In SL, identity is treated as sortal-relative, i.e. the basic notion of identity is relativized to a given sortal, so $a =_S b$ means that a is the same S as b (where "S" is a variable ranging over sortals). The classical identity $a = b$ is taken as an enlargement of the original sortal one.

The notion of difference can be in SL simply defined as $a \neq_S b$, a is different S than b. But, in AL the notion of difference is something diverse. In Porphyry, in his *Introduction to Aristotle's Catogories*, we can read that difference is "that by which a species exceeds its genus". I will use this characteristics and well known Aristotle's definition of species as a combination of superordinate genus and specific difference. For reconstruction of AL, I will suppose for simplicity that sorts can be roughly identified with Aristotelian's species. Now, sortal can be characterised as a set Φ of concepts. Φ consists of two concepts, superordinate sortal (genus proximum) α and specific difference (differentia specifica) β. α and β are of a very diverse nature; α is a sortal, so connected with a principle of identity, i.e. according to Strawson (1959) "a principle for distinguishing and counting individual particulars", whereas β is – in Geach (1962) terminology – an adjectival term. As Dummett (1973) suggests, sortal is also connected with a criterion of application, that which determines when it is correct to apply a predicate to an individual. Most adjectives, according to Dummett, are connected only with a criterion of application. This is also a case of β, it shares the same criterion of application as "its" α. So, specific differences are always genus-relative, i.e. are meaningfully applied only to the members of the given superordinated sortal (= genus).

The notion of sortal identity for sortally interpreted AL (SAL) can be then defined as $\forall x \forall y(x =_S y \equiv \exists \Phi(\Phi x \equiv \Phi y))$. The corresponding notion of sortal difference for SAL can be defined as $\forall x \forall y(x \neq_S y \equiv \exists \Phi(\Phi x \wedge \neg \Phi y))$. A terminological remark – sortal difference is taken to mean some relation between objects. Traditionally, specific difference is taken to mean some monadic predicate (rational, animated atc.). So, sortal difference is a kind of binary predicate, specific dfference is a kind of unary predicate. Hence, specific difference is the cause of sortal difference. It seems that we can simply say, e.g. Socrates and Bucephalus are sortally different, then it means that Socrates is a man and Bucephalus is not. But, traditionally in SL negation of a sortal is not a sortal. According to the proponents of SL, "is not a cat" is not a sortal (Wiggins, 1980), because you cannot count the non-cats since they include dogs, tables, molecules etc., including even the uncountable terms. Obviously, here the negation of a sortal means an external negation. So, I will use the abovementioned conception of sortal as the set Φ of superordinate sortal α and specific difference β and say that "Socrates is a man and Bucephalus is not" means that Socrates and Bucephalus shares the common sortal, namely animal, and that the Aristotelian difference "rational" holds for Socrates but is negated for Bucephalus. Non-rational animals can be counted, so the very concept of non-rational provides a criterion for counting, thus it is a sortal. So, generally internal negation of a sortal S, say $\sim S$, is a sortal which consists in a set of properties Φ'. Φ' includes the same α as S, but contains negation of β. If by negation of a sortal we mean external (Boolean) negation of a sortal (negation of the whole Φ), then the result is not a sortal.

Moreover, we can now say that for any given sortal $\Phi : (\alpha, \beta)$, when β is negated, then $\Phi : (\alpha, \sim\beta)$ is internal negation of $\Phi : (\alpha, \beta)$, e.g. non-smoker is not a smoker, but is still a man. If the whole set (α, β) is negated, then this negation is external negation of $\Phi : (\alpha, \beta)$. Further, if we stay within the given α, then to affirm or to deny any appropriate β preserves the principle of excluded middle, so in the "sphere" of α negating of β's behaves like external negation. Similarly, if α is "object" or "entity" and there is no other sortal whatsoever, then to affirm or to deny any β preserves the principle of excluded middle. So, the very distinction between external and internal negation takes its place when we talk about different sorts of objects. This is often the case of natural language (where we can thus find the internal negation), but is not the case of first-order logic (where the very distinction is not easy to introduce). In AL (internal) negation of a concept is clearly very important feature, especially in the case of negating differences. So, with the help of the previous definitions, the concept of difference and its negation can be easily implemented into SAL.

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Negation on the neo-Australian Plan

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A popular contemporary view of negation understands negation to be an intensional connective underpinned by a two-place, symmetric (in)compatibility relation. I show here that there are sensible directionally sensitive intensional negations that are underpinned by a *non-symmetric* (in)compatibility relation.

Classical negation is an *extensional* one-place connective insofar as the truth-value of $\sim A$ at some point x in a given model $\mathbf{M}$ depends simply on the truth-value of the negand A at that same point. That is, $x \vDash \sim A \Leftrightarrow x \nvDash A$. By contrast, an *intensional* conjunction will be one where the truth-value of $\sim A$ at some point x in a given model $\mathbf{M}$ depends on the truth-value of A at some *other* points $y, z \ldots$ in the model, such that those points are related to x in some special manner or other. Any one-place intensional connective is a member of the family of modalities, the most well known of which is the familiar $\Box$-operator from modal logics such as S5 and so forth.

The Australian Plan for negation (Berto and Restall, 2018) takes negation to be a one-place intensional connective, a modality of a particular sort. This negation is introduced via a *compatibility frame*, $\mathbf{F_C} : \langle S, \sqsubseteq, C \rangle$, where S is a set of information states $x, y, z, \ldots$, $\sqsubseteq$ is a partial order on the members of S, and C is a binary compatibility relation on members of S such that xCy means that the information carried by x is compatible with the information carried by y.[1] C and $\sqsubseteq$ interact as we would expect:

$$\text{If } x \sqsubseteq y \text{ and } yCz, \text{ then } xCz \tag{1}$$

This makes sense. If the information carried by x is contained within the information carried by y, and the information carried by y is compatible with the information contained in z, then the information carried by x is compatible with the information carried by z also.

Writing $x, y \ldots \in \mathbf{F_C}$ as an abbreviation for $x, y \cdots \in S$ where $S \in \mathbf{F_C}$, and reading $x \Vdash A$ as "the information state x carries information of type A", we can use our compatibility relation to get the following model theoretic condition for our intensional negation $\sim A$:

$$x \Vdash \sim A \textit{ iff } \text{for each } y \in \mathbf{F_c} \text{ s.t. } xCy, y \nVdash A \tag{2}$$

This makes sense too. x will carry the information that $\sim A$ just in case for any information state y such that x is compatible with y, y will not carry the information that A.

According to the Australian plan for negation, *(in)compatibility is symmetric* (Berto, 2015), (Berto and Restall, 2018), (Restall, 1999).

> Whatever kind of entities a and b are, it seems that if a is (in)compatible with b then b is (in)compatible with a (Berto and Restall, 2018), p. 21.

[1] The same negation may be introduced via an *incompatibility*, or perp frame $\langle S, \sqsubseteq, \perp \rangle$, where $\perp$ is a binary incompatibility relation on members of S (Dunn, 1994). Now $\sim A := \{X : A \perp X\}$, with $x \perp y$ being read as *the information carried by* y *is incompatible with the information carried by* y. Translation between compatibility frames and perp frames is straightforward on account of $\forall x, y((xCy) \Leftrightarrow (x \not\perp y))$. Although the entire argument given above and below can be given in terms of incompatibility just as well as it can be given in terms of compatibility, we will restrict ourselves to the latter.

According to the neo-Australian plan for negation that I shall be articulating and defending here, *(in)compatibility is non-symmetric* (neither symmetric nor asymmetric). There are cases where a is (in)compatible with b, but b is not (in)compatible with a.

To motivate the neo-Australian plan, note that negations of any type will have it in common that, at the very least, they are ruling *something* out, see p. 4 of (Berto and Restall, 2018), along with (Sequoiah-Grayson, 2009), and (Sequoiah-Grayson, 2010). Consider the familiar Boolean negation from classical propositional logic. Such a negation is *ruling out truth*. Indeed, this is exactly what it is that is specified by the semantic clause for classical negation, namely that $x \vDash {\sim} A \Leftrightarrow x \nvDash A$. In the classical case, the ruling out of truth will permit falsity by definition, but the mere act of the ruling out of truth does not imply this strictly, consider constructive negations originating from Intuitionistic logic for example. What classical and constructive negations have in common is that they are *static* in that they do not rage over, that is rule out, dynamic processes, procedures, or actions. By contrast, a dynamic negation *will* rule out processes, procedures, or actions, whatever these might be. It is by exploring dynamic negations in detail that the non-symmetry of compatibility becomes apparent.

Dynamic negations turn up in many places, one of which is within recent substructural approaches to epistemic logic.[2] In particular, they appear when we consider *epistemic actions* explicitly. In what follows, I shall show that it is within the context of fine-grained psychological epistemic actions that dynamic negations, along with a corresponding failure of compatibility, find one of their more philosophically interesting homes.

We start with an information frame $\mathbf{F_I} : \langle S, \sqsubseteq, \bullet \rangle$. S and $\sqsubseteq$ are as before, and $\bullet$ is a binary composition order on members of S (Restall, 1996), (Dunn and Hardegree, 2001). We interpret information inclusion as *information relevance*, so $x \sqsubseteq y$ is read as *the information carried by* x *is relevant to the information carried by* y, Dunn (2015). $x \bullet y \sqsubseteq z$ is read as *the combination of the information carried by* x *with the information carried by* y *is relevant to the information carried by* z. We use this dynamic information construction to give semantic conditions for intensional connectives. Consider intensional implication, $\rightarrow$:

$$x \Vdash A \rightarrow B \textit{ iff } \forall y, z \text{ s.t. } x \bullet y \sqsubseteq z, \text{ if } y \Vdash A \text{ then } z \Vdash B. \tag{3}$$

Given the information carried by the states x and y, their combination will be relevant to the information carried by z. We have an *operational semantics* here on account of the dynamic combinatorial operations on information states sitting centre stage, (Sequoiah-Grayson, 2016).

To introduce our dynamic negation we combine the intensional implication above with bottom, $\mathbf{0}$:

$$x \Vdash \mathbf{0} \text{ for no } x \in \mathbf{F_I} \tag{4}$$

We can now define our dynamic negation as $A^{\mathbf{0}} := A \rightarrow \mathbf{0}$, which gives us the following evaluation condition for $A^{\mathbf{0}}$:

$$x \Vdash A^{\mathbf{0}} \textit{ iff } \forall y, z \text{ s.t. } x \bullet y \sqsubseteq z, \text{ if } y \Vdash A \text{ then } z \Vdash \mathbf{0}. \tag{5}$$

Our dynamic negation is not ruling out truth, it is ruling out a certain procedure, namely $x \bullet y$. It is negation as procedural prohibition (Sequoiah-Grayson, 2009). $A^{\mathbf{0}}$ is the type of information that can never be combined with information of type A, on pain of such a combination failing to result in any information at all.

[2]Consider also the pragmatic context within which one performs the speech act of asserting of denying a claim, (Restall, 2013). Such an act is not to merely rule out the truth of the proposition being denied, but to rule out the assertion of that same proposition. Similarly, the speech act of assertion rules out the act of denial.

The relevant question for our purposes is this - will $\mathbf{F_I}$ commute? That is, will we have it that $\forall x, y, z(x \bullet y \sqsubseteq z \Rightarrow y \bullet x \sqsubseteq z)$? The answer will depend on our interpretation of the domain.

We give $\mathbf{F_I}$ a robust epistemic interpretation such that the members of S are states of explicit knowledge of some agent α, and $\bullet$ marks the psychological epistemic action of combining the objects of epistemic propositional attitudes. Under this epistemic interpretation, $\sqsubseteq$ has moved from bare information relevance to *epistemic relevance*.[3] Here, commutation will fail, and so to will (in)compatibilty.[4]

Suppose that $x \Vdash p \to q$, $y \Vdash q \to r$, and $z \Vdash p \to r$. In this case $x \bullet y \sqsubseteq z$, but $y \bullet x \not\sqsubseteq z$ Unlike the former, the latter epistemic action is not epistemically relevant to α's being in state z at all. $x \bullet y$ is the correct order of combination insofar as cutting q is concerned, whereas $y \bullet z$ is the wrong order insofar as it is an attempt to give r to an input the accepts p.[5]

The operational clause $x \bullet y \sqsubseteq x$ and the compatibility relation C are related closely:

$$xCy \text{ iff } \exists z(x \bullet y \sqsubseteq z) \tag{6}$$

Things are now in place! In our example above, we have it that $\exists z(x \bullet y \sqsubseteq z)$, but we do *not* have it that $\exists z(y \bullet x \sqsubseteq z)$ on account of $y \bullet x \not\sqsubseteq z$. In fact there is no epistemic state resulting from $x \bullet y$ at all.[6] Hence xCy but *not* yCx. Although x carries information of a type that can be combined with information of the type carried by y, x does not carry the type of information that can have information of the type carried by y combined with *it*.[7]

The typing involved here is directionally/order sensitive, so we need to mark this with a directional dynamic negation. Given that commutation fails under our epistemic interpretation of $\mathbf{F_I}$, we will mark directional difference with the addition of a right-to-left arrow:

$$x \Vdash B \leftarrow A \text{ iff } \forall y, z \text{ s.t. } y \bullet x \sqsubseteq z, \text{ if } y \Vdash A \text{ then } z \Vdash B. \tag{7}$$

We will use the left-arrow from (7) along with $\mathbf{0}$ to get a new negation ${}^{\mathbf{0}}A := \mathbf{0} \leftarrow A$ such that:

$$x \Vdash {}^{\mathbf{0}}A \text{ iff } \forall y, z \text{ s.t. } y \bullet x \sqsubseteq z, \text{ if } y \Vdash A \text{ then } z \Vdash \mathbf{0} \tag{8}$$

We now have not one, but two dynamic negations, a *split negation pair*, $A^{\mathbf{0}}$ and ${}^{\mathbf{0}}A$. Given their respective evaluation conditions (5) and (8), some proposition, ϕ will be of type $A^{\mathbf{0}}$, written $\phi : A^{\mathbf{0}}$, just in case ϕ can never be applied to information of type A, whereas $\phi : {}^{\mathbf{0}}A$ just in case ϕ can never have information of type A applied to it.

Concretely and in sum, it is the case that $(q \to r) : (p \to q)^{\mathbf{0}}$, but it is *not* that case that $(p \to q) : (q \to r)^{\mathbf{0}}$.[8] On the space of propositional attitudes with a combinatorial operation of

[3]See (Aucher, 2014), (Aucher, 2015), (Bilkova, Majer, and Pelis, 2015), and (Sedlar, 2015) for recent substructural approaches to epistemic logics in general, and (Sequoiah-Grayson, 2016) for an interpretation of informational relevance in terms of epistemic relevance in particular.

[4]It is important to note that we will not have (in)compatibilty failure *because* of commutation failure. Commutation failure on an operation might be achieved when that operation is merely order *sensitive*, but will still give *some* output when the order of combination is switched. Consider subtraction and division on the domain of integers as examples. Commutation will fail for our epistemic action operation precisely *because* (in)compatibilty is non-symmetric across the domain of propositional attitudes.

[5]At the syntactic level with intensional conjunction, $(q \to r) \otimes (p \to q)$ is the wrong order insofar as combining dynamic information is concerned.

[6]Importantly, it is not the case that $x \bullet y$ will result in some epistemic state of α such that this state carries the information that $(q \to r) \wedge (p \to q)$. To think this would be to confuse mere *knowledge aggregation* with *knowledge composition*. *Ipso facto* for any other attitudinal actions, doxastic or otherwise.

[7]Given that direction is now a distinction with a difference, 'application' might be a better term that 'combination'.

[8]This is on account of the fact that, as we have seen, syntactically speaking we have it that $(p \to q) \otimes (q \to r) \vdash (p \to r)$.

psychological epistemic actions, such operations will be delicately order-sensitive. So delicate that some reorderings may, as we have seen, block epistemic progress to any further epistemic state whatsoever. Hence the (in)compatibility of such states or attitudes is non-symmetric.[9]

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[9]Moreover, once we start typing objects functionally, we find non-symmetric (in)compatibility relations all over the place. For example, directional informational (in)compatibility is mark of natural languages, and as such the relevant lack of symmetry will find its representation in functional analyses of linguistic phenomena. For example, 'Alexandra scampers' is well-formed, whereas 'scampers Alexandra' is not. Lambek Calculi and their cognate logics have their non-commuting fragments for just this reason. Unsurprisingly, (in)compatibility relations are non-symmetric across the same phenomena (Sequoiah-Grayson, 2010).

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Validity for Alethic Pluralists

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It has been argued that alethic pluralists—who hold that there are several distinct truth properties—face a problem when it comes to defining validity. This paper presents two new solutions to the problem. This should be of interest not only to alethic pluralists but to philosophical logicians and philosophers of logic more generally—because the key to the second solution is a proper understanding of the classical notion of validity.

The challenge for pluralists is presented by Tappolet (1997, 209–10) as follows:

> Consider the following inference:
>
> (1) Wet cats are funny.
> (2) This cat is wet.
> *Ergo*, this cat is funny.
>
> The validity of an inference requires that the truth of the premisses necessitates the truth of the conclusion. But how can this inference be valid if we are to suppose... that two different kinds of truth predicates are involved in these premisses? For the conclusion to hold, some unique truth predicate must apply to all three sentences. But what truth predicate is that? And if there is such a truth predicate, why isn't it the only one we need?

The problem arises with *mixed inferences* where at least two of the component propositions (premisses and/or conclusion) are in the domains of distinct truth properties. Some such inferences appear to be valid—and some do not. The challenge for the pluralist is to define validity in such a way as to maintain these appearances, without departing (too radically) from the classical understanding of validity as involving necessary truth preservation.

A natural thought for the pluralist is to turn to many-valued logics (MVL) for leads on how to define validity when we have multiple kinds of truth. This is the approach taken by Beall (2000, 382): "an argument is valid iff (necessarily) if all the premises are designated, then the conclusion is designated." Beall's approach has however faced the objection that 'having a designated value' looks like a generic truth property—whereas the challenge is to define validity *without* appealing to a generic notion of truth.

In fact there are at least three standard ways of defining validity in MVL:

1. Pick a single one of the truth values and define validity in terms of preservation of that value.

2. Pick a subset of the truth values as designated values and define validity in terms of preservation of designation.

3. Define an ordering on the truth values and define validity in terms of that ordering.

Beall followed the lead of the second approach. I shall give two new definitions of validity for alethic pluralists: one that follows each of the other two approaches.

I first pursue the third option and give a way of defining validity that will work both for alethic pluralists who think that each individual proposition has exactly one kind of truth property and for alethic pluralists who think that some propositions may possess multiple truth properties at the same time.

I then turn to the first approach: picking *one* of the truth values and defining validity in terms of preservation of that value. This option might seem to be a non-starter because in mixed inferences there is no single value that can be possessed by all the propositions in the argument. I argue that this is a mistake and that validity *can* be defined in terms of preservation of a single value—even if it is a value that not all the propositions in a given mixed inference can possess. Furthermore, once we can do this for one of the truth values, we can do it once for each of them, yielding multiple notions of validity—one for each kind of truth—which seems like a natural option for an alethic pluralist. The key to making this idea work is recalling that the classical notion of validity involves *two* ideas: necessary truth preservation, in virtue of form. Many contributors to the literature on validity and alethic pluralism seem to overlook the second idea (in virtue of form) and focus only on the first (necessary truth preservation):

> the Tarskian idea that validity is necessary truth-preservation (Beall, 2000, 381)
>
> the classical account of validity, according to which an argument is valid on condition that the truth of the premises necessitates the truth of the conclusion. (Tappolet, 2000, 383)
>
> Validity is standardly understood in terms of necessary truth preservation: necessarily, if the premises are true, so is the conclusion. (Pedersen, 2012, 592)
>
> the standard characterization of validity as necessary truth preservation (Cotnoir, 2013, 565)

I argue that once we recall the role of *form* in the definition of validity, the way is open to defining validity in terms of preservation of a *single* notion of truth.

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Proof-Theoretic Semantics and the Interpretation of Atomic Sentences

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In the proof-theoretic semantical systems of Nissim Francez, introduction and elimination rules for the logical operators determine the semantic values of logically complex sentences. Differences in the rule systems the logician employs (e.g. intuitionistic and classical) correspond to differences in the semantic values of the sentences interpreted through those systems. In this regard the proof-theoretic semanticist makes good on the claim that classical and intuitionistic logic correspond to different conceptions of meaning. To date, however, there has been little discussion of the semantic values of atomic sentences in proof-theoretic semantics (PTS). Their values are either assumed given from outside the proof system (e.g. (Francez, Dyckhoff, and Ben-Avi, 2010), (Francez, 2016), and (Wieckowski, 2011), or stipulated via definitional systems of the sort found in (Prawitz, 1973). In this essay I consider some candidate introduction and elimination rules for atomic sentences, and I investigate the conception of linguistic meaning that is induced by proof systems of this sort.

In PTS the semantic values of sentences are delimited by their canonical derivations. Roughly, a derivation *to* some logically complex sentence A is I-canonical just in case it is one whose conclusion is A and whose final step is justified by appeal to an introduction rule for the major operator of A. A derivation *from* some logically complex sentence A is E-canonical just in case the first rule applied to A as a major premise in the derivation is an elimination rule for the major operator in A. Canonical derivations determine what (Francez, 2015) calls a sentence's proof-theoretic denotation at a context.

One proposal for supplying introduction and elimination rules for atomic sentences, and so specifying an intended interpretation for an atomic system, looks to model theory. Pos-sible worlds semantics offers an intuitive conception of introduction rules for atomic sen-tences insofar as they specific the elementary states of affairs that atomic sentences purport to signify. Similarly, with a planning-based expressivist semantics of the sort developed by (Gibbard, 2003) and (Stovall, Forthcoming), at least some atomic sentences can be associated with model-theoretic elimination rules concerning how agents should or shall act. It might be thought that a model-theoretic framework for studying the meanings of atomic sentences is not sufficiently proof-theoretic in spirit, however.

Another possibility is to use explanatory inferences so as to specify an intended interpretation of atomic sentences. Just as there are both introduction and elimination rules, so are their two orders or directions of explanation. On one hand we may keep a sentence fixed and look to see which contexts or circumstances better explain it. On the other hand we may consider sentences across different contexts and look to see which other sentences are bet-ter explained by it. With a particular conception of explanation (e.g. deductive-nomological, causal-mechanical, unificationist, or material) we define atomic introduction and elimination rules as follows: an introduction rule for an atomic sentence p specifies the contexts G that explain p, while an elimination rule for an atomic sentence p specifies the context/sentence pairs $\langle G, q\rangle$ such that p together with G explains q. Proof-theoretic denotations for atomic sentences are then given by the sets of I- and E-canonical derivations in which they figure, just as with the logically complex sentences. The result is a unified semantics for atomic and logically complex sentences sufficient to individuate an intended interpretation for the former.

Much as the choice between an intuitionistic and a classical set of logical rules induces different conceptions of the meanings of logically complex sentences, so do variations in the notion of explanation induce different notions of atomic meaning. Explanations are in general non-monotonic, and non-monotonic explanatory inferences can be reconstructed proof-theoretically (see (Litland, 2017) and (Millson, Khalifa, and Risjord, 2018)). On the other hand, those who favor monotonic derivation systems can appeal to a deductive-nomological account of explanation of the sort pioneered by (Hempel, 1965). The former corresponds to a context-variant notion of meaning whereas the latter corresponds to a notion of 'core' or 'essential' meaning.

Proof-theoretic notions of meaning also offer productive standpoints from which to investigate different areas of philosophy and linguistics that have been dominated by model-theoretic semantical systems. For instance, notice that whereas A and A&A are true at all and the same possible worlds, the proof theorist can distinguish their semantic values: for the derivations to and from these sentences will differ. This additional fineness of grain allows the proof theorist to account for the semantic values of so-called 'hyperintensional' operators without having to appeal to a model with metaphysical postulates. Just so, with a PTS for atomic sentences in terms of their roles in explanation, the semantic values of sentences employing hyperintensional operators like 'in virtue of' and 'essentially' can be calculated on the basis of these explanations. It follows that object-language debate about metaphysical features of the world can be understood as covertly metalinguistic debate concerning how best to reason about the world, a view rooted in the foundations of proof theory (e.g. in the work of (Carnap, 1934)).

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Validity of Inference versus Validity of Demonstrations

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Every logician knows, or at least should know, how to classify inference (-figures, schemata) into valid and invalid ones. Today this commonly gets done by reducing the validity of the inference in question to the logical truth ("in all variations") of a certain implicative proposition, following Wittgenstein in *TLP*, or Quine in *Methods of Logic*, or, equivalently, following Bolzano, to the logical holding of consequence (*Ableitbarkeit*) from antecedent propositions to the consequent proposition. Validity of inference in a *qualifying* notion.

When we demonstrate a mathematical theorem, after its demonstration has been concluded, thats it; we do not then have to demonstrate that the demonstration is valid. *Post factum*, it might happen that the line of thought as given is found insufficiently clear, or indeed plain wrong. Doubts might be raised as to the validity of the theorem and this calls for a meticulous examination of all the axioms and inferences used in the demonstration as given: are the axioms really self-evident and are all the inferences used really valid ones? If one of these questions will have to be answered in the negative, the theorem will have to be retracted: its demonstration turned out to be *invalid*.

Invalid demonstration can best be compared to *false friend*. Also in the case of friends, true/false are not qualifying notions. A false friend has been removed from the category of friends. However, the modifying notion *false* has got a matching *restorative* (or *restitutive*) notion, namely *true*: we might after proper inquiry found that the alleged *false friend* was after all a *true friend*. After the same fashion, it might turn out that an allegedly *invalid* demonstration was in fact, a *valid* demonstration.

The talk attempts to spell out the difference between the qualifying notion of validity of inference and the modifying/restitutive notions of invalidity/validity of demonstration.

Cut-free and Analytic Sequent Calculus of Intuitionistic Epistemic Logic

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[Artemov and Protopopescu, 2016] gave an intuitionistic epistemic logic based on a verification reading of the intuitionistic knowledge in terms of Brouwer-Heyting-Kolmogorov interpretation. According to this interpretation, a proof of $A \supset B$ is a construction such that when a proof of A is given, a proof of B can be constructed. [Artemov and Protopopescu, 2016] proposed that a proof of a formula KA (read "it is known that A"), is the conclusive verification of the existence of a proof of A. Then $A \supset KA$ expresses that, when a proof of A is given, the conclusive verification of the existence of the proof of A can be constructed. Since a proof of A itself is the conclusive verification of the existence of the proof of A, they claim that $A \supset KA$ is valid. But $KA \supset A$ (usually called *factivity* or *reflection*) is not valid, since the verification does not always give a proof. They provided a Hilbert system of intuitionistic epistemic logic **IEL** as the intuitionistic propositional logic plus the axioms schemes $K(A \supset B) \supset KA \supset KB$, $A \supset KA$ and $\neg K\bot$. Moreover they gave **IEL** the following Kripke semantics. We say that $M = (W, \leq, R, V)$ is a *Kripke model* for **IEL** if $(W, \leq, V)$ is a Kripke model for intuitionistic propositional logic and R is a binary relation such that $R \subseteq \leq$, $\leq ; R \subseteq R$ and R satisfies the seriality. Then KA is true on a state w of M if and only if for any v, wRv implies A is true in v of M. Let $\mathbb{M}_{\mathbf{IEL}}$ be the class of all Kripke models and $\mathbb{M}_{\mathbf{IEL}}^{\text{finite}}$ the class of all finite Kripke models. [Artemov and Protopopescu, 2016] also proved that their Hilbert system is sound and complete for $\mathbb{M}_{\mathbf{IEL}}$. As far as the authors know, it is not known yet if their Hilbert system is complete for $\mathbb{M}_{\mathbf{IEL}}^{\text{finite}}$, i.e., the system enjoys the finite model property.

The study of **IEL** also casts light on the study of the knowability paradox. The knowability paradox, also known as the Fitch-Church paradox, states that, if we claim the knowability principle: every truth is knowable ($A \supset \Diamond KA$), then we are forced to accept the omniscience principle: every truth is known ($A \supset KA$) [Fitch, 1963]. This paradox is commonly recognized as a threat to Dummett's semantic anti-realism. It is because the semantic anti-realists claim the knowability principle but they do not accept the omniscience principle. However, as Dummett admitted that he had taken some of intuitionistic basic features as a model for an anti-realist view [Dummett, 1978, p.164], it is reasonable to consider an intuitionistic logic as a basis. In this sense, if we employ BHK-interpretation of KA as above to accept the **IEL** in the study of the knowability paradox, $A \supset KA$ becomes valid and the knowability paradox is trivialized.

[Krupski and Yatmanov, 2016] provided a sequent calculus of **IEL**. The sequent calculus is obtained from Gentzen's sequent calculus **LJ** (with structural rules of weakening and contraction) for the intuitionistic logic plus the following two inference rules on the knowledge operator:

$$\frac{\Gamma_1, \Gamma_2 \Rightarrow A}{\Gamma_1, K\Gamma_2 \Rightarrow KA}\,(KI) \qquad \frac{\Gamma \Rightarrow K\bot}{\Gamma \Rightarrow F.}\,(U)$$

where a sequent $\Gamma \Rightarrow A$ (where Γ is a finite multiset of formulas) can be read as "if all of Γ hold then A holds." They established the cut-elimination theorem of the calculus, i.e., a derivable sequent in their system is derivable without any application of the following cut rule:

$$\frac{\Gamma \Rightarrow B \quad B, \Sigma \Rightarrow \Delta}{\Gamma, \Sigma \Rightarrow \Delta} \ (Cut),$$

where Δ contains at most one formula. They also specify the computational complexity of **IEL** as PSPACE-complete by a proof-theoretic method in terms of their sequent calculus. It is remarked, however, that this system does not enjoy the subformula property. That is, in the rule of (U), we have a formula $K\bot$ which might not be a subformula of a formula in the lower sequent of the rule (U).

This talk gives a new cut-free and analytic sequent calculus $\mathscr{G}(\mathbf{IEL})$ of the intuitionistic epistemic logic, which is obtained from adding the following rule (K_{IEL}) into Gentzen's **LJ**:

$$\frac{\Gamma_1, \Gamma_2 \Rightarrow \Delta}{\Gamma_1, K\Gamma_2 \Rightarrow K\Delta} \ (K_{IEL})$$

where Δ contains at most one formula. It is easy to see that (K_{IEL}) satisfies the subformula property. Then we show that the new system is equivalent to the system above from [Krupski and Yatmanov, 2016]. In the new system the rule of (KI) is admissible obviously and the admissibility of the rule (U) can be shown as follows:

$$\cfrac{\cfrac{\Gamma \Rightarrow K\bot \qquad \cfrac{\bot \Rightarrow}{K\bot \Rightarrow}\ (K_{IEL})}{\Gamma \Rightarrow}\ (Cut)}{\Gamma \Rightarrow F,}\ (RW)$$

where (RW) is the right rule of weakening. On the other hand, the rule (K_{IEL}) is also shown to be admissible in the old system. That is, if Δ is a singleton in the rule (K_{IEL}), then the rule is the same as (KI). If Δ is empty, we have the following derivation:

$$\cfrac{\cfrac{\cfrac{\cfrac{\Gamma_1, \Gamma_2 \Rightarrow}{\Gamma_1, \Gamma_2 \Rightarrow \bot}\ (RW)}{\Gamma_1, K\Gamma_2 \Rightarrow K\bot}\ (K_{IEL})}{\Gamma_1, K\Gamma_2 \Rightarrow \bot}\ (U) \qquad \bot \Rightarrow}{\Gamma_1, K\Gamma_2 \Rightarrow}\ (Cut)$$

Let $\mathscr{G}^{-c}(\mathbf{IEL})$ be the system $\mathscr{G}(\mathbf{IEL})$ without the cut rule. By the standard syntactic argument, we can establish the following fundamental proof-theoretic result.

Theorem 1 (Cut-Elimination) *If* $\mathscr{G}(\mathbf{IEL}) \vdash \Gamma \Rightarrow \Delta$ *then* $\mathscr{G}^{-c}(\mathbf{IEL}) \vdash \Gamma \Rightarrow \Delta$.

Corollary 1 (Disjunction Property and Craig Interpolation Theorem)
1. *If* $\Rightarrow A \vee B$ *is derivable in* $\mathscr{G}(\mathbf{IEL})$, *then either* $\Rightarrow A$ *or* $\Rightarrow B$ *is derivable in* $\mathscr{G}(\mathbf{IEL})$.
2. *If* $\Rightarrow A \supset B$ *is derivable in* $\mathscr{G}(\mathbf{IEL})$, *then there exists a formula* C *such that* $\Rightarrow A \supset C$ *and* $\Rightarrow C \supset B$ *are also derivablein* $\mathscr{G}(\mathbf{IEL})$ *and all propositional variables of* C *are shared by both* A *and* B.

This corollary follows from the cut-elimination theorem since all inference rules except the cut rule satisfy the subformula property in our system. It is noted that the rule (U) in the system from [Krupski and Yatmanov, 2016] seems to cause a difficulty in establishing the disjunction property.

Let us say that $\mathscr{G}(\mathbf{IEL})$ enjoys the *finite model property* if $\mathbb{M}^{\text{finite}}_{\mathbf{IEL}} \models \Gamma \Rightarrow \Delta$ implies $\mathscr{G}(\mathbf{IEL}) \vdash \Gamma \Rightarrow \Delta$ for all sequents $\Gamma \Rightarrow \Delta$, where $\mathbb{M} \models \Gamma \Rightarrow \Delta$ means that, for every finite model (i.e., from $\mathbb{M}^{\text{finite}}_{\mathbf{IEL}}$) and for every state in this model, if all formulas A in Γ is true in the state then there is a formula B in Δ such B is true in the state. We establish the finite model property in an informative way, i.e., our proof also gives us an alternative semantic proof of the cut-elimination theorem of $\mathscr{G}(\mathbf{IEL})$. A key lemma, the cut-free completeness of $\mathscr{G}^{-c}(\mathbf{IEL})$, is the following, from which we can easily obtain the finite model property of $\mathscr{G}(\mathbf{IEL})$.

Lemma 1 *If* $\mathbb{M}^{\text{finite}}_{\mathbf{IEL}} \models \Gamma \Rightarrow \Delta$ *then* $\mathscr{G}^{-c}(\mathbf{IEL}) \vdash \Gamma \Rightarrow \Delta$.

This lemma is proved by a combination of an argument in [Hermant, 2005] for intuitionistic logic and an argument in [Takano et al., 2018] for modal logic. Since it is easy to prove that $\mathscr{G}(\mathbf{IEL})$ (with the cut rule) is sound for $\mathbb{M}^{\text{finite}}_{\mathbf{IEL}}$, we can also derive Theorem 1 from Lemma 1.

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Do We Need Recursion?

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Recursion in a narrow sense is an operation with (multivariable) functions that process natural numbers. That is, we say that a function f is derived from two functions g and h using *primitive recursion* if the following two equations hold for any choice of arguments: $f(0,\underline{y}) = g(\underline{y})$ and $f(x+1,\underline{y}) = h(f(x,\underline{y}),x,\underline{y})$. In these equations, $\underline{y}$ stands for the k-tuple $y_1,..,y_k$ and g, h and f are functions of k, $k+2$ and $k+1$ variables respectively. A variant of primitive recursion is *course-of-values recursion* for which the defining equation is $f(x,\underline{y}) = h(\tilde{f}(x,\underline{y}),\underline{y})$ where $\tilde{f}(x,\underline{y})$ denotes the numerical code of the sequence $f(0,\underline{y}),..,f(x-1,\underline{y})$. To determine $f(x,\underline{y})$ for a function f derived by course-of-values recursion, several or all values of f for arguments smaller than x might be needed. Since course-of-values recursion can be derived from primitive recursion, it is not really more powerful; but it is often very convenient. Primitive recursion appears in very basic definitions: a function is *partial recursive* if it can be derived from the initial functions using composition, minimization and primitive recursion; it is *primitive recursive* if it can be derived from the same initial functions using composition and primitive recursion only. Since *RE* sets are usually defined as domains of partial recursive functions, the operation of primitive recursion is in fact part of the definition of the arithmetical hierarchy.

Recursion in a broader sense is used in programming languages: a procedure can be written so that it processes its parameter by calling itself, perhaps several times, with parameters that are simpler in some sense. The parameters do not have to be natural numbers. In logic we have several definitions that are described as recursive. One example is this: an expression is a term if it is a variable, or if it is a constant, or if it has the form $F(t_1,..,t_n)$ where $t_1,..,t_n$ are terms. This and other syntactic definitions deal with strings rather that numbers. However, if syntactic objects are identified with numbers, all these definitions appear to be applications of course-of-values recursion.

When dealing with metamathematics of Peano arithmetic PA and with incompleteness phenomena, one might need arithmetic formulas that define *RE* sets. When dealing with Gödel's second incompleteness theorem, one (of course) needs logical syntax formalized inside PA. In both situations primitive recursion poses a problem because in the language of arithmetic (consisting of the binary operation symbols $+$ and $\cdot$, the order symbols $<$ and $\leq$, the constant 0 and the successor function S) there is nothing that would directly correspond to it. One can use existential quantifiers and describe a function that is derived by composition (from functions that can be described), and one can use the least number principle to describe a function that is derived by minimization. However, the dynamic nature of primitive recursion is problematic for the language of a formal theory where we primarily have static descriptions. S. Feferman in his paper [1], which for decades was the most important source of information about Gödel's theorems and about interpretability, introduces the notion of PR-formulas. The purpose of this notion is to have a class of formulas that describe exactly the primitive recursive conditions. Nevertheless, PR-formulas are more an *ad hoc* technical solution than wisely chosen notion that can be further studied. This observation is where the title of this talk emerges.

Polynomials (multivariable polynomials in the domain N of natural numbers) are functions like $[x,y] \mapsto 2x^2 + 3xy + 1$, obtained via composition from addition, multiplication and constants. A *bounded condition* (or Δ_0 *condition*) is a condition obtained from equalities of polynomials using Boolean operations and the *bounded quantifiers* $\forall v{\le}f(\underline{x})$, $\exists v{\le}f(\underline{x})$, $\forall v{<}f(\underline{x})$ and $\exists v{<}f(\underline{x})$ where f is a polynomial not dependent on v. The meaning of bounded quantifiers is obvious. For example, $\forall v{<}f(\underline{x})A(v,\underline{x})$ means $\forall v(v < f(\underline{x}) \Rightarrow A(v,\underline{x}))$. Many frequently needed conditions can be classified as Δ_0: x is a divisor of y (in symbols, $x \mid y$) if $\exists v{\le}y\,(v \cdot x = y)$, and x is prime if $x \neq 1$ & $\forall v{<}x\,(v \mid x \Rightarrow v = 1)$. Euclidean division, i.e. the two functions Mod and Div that yield the remainder and the quotient of dividing x by y, have Δ_0 graphs.

It is not so difficult to prove that *RE* sets are exactly all projections (conditions obtained by existential quantification) of Δ_0 relations. Also, if the list of initial functions is slightly extended (by adding addition, multiplication and the characteristic function of the equality relation), primitive recursion can be removed from the definition of partial recursive functions. These facts are proved or at least suggested in [3]. Defining partial recursive functions without mentioning primitive recursion might look paradoxical, but it is actually convenient. The claim that all *RE* sets are definable in the structure $\mathbb{N}$ of natural numbers, which is an essential ingredient of structural proofs of first incompleteness theorem, is then very easy to prove.

We will show that logical syntax can be arithmetized without using primitive recursion. This means that also Gödel's second incompleteness theorem is by no means dependent on primitive recursion. As an example, take the notion of *term* in the language of arithmetic: since we have a context-free grammar for this notion, a well-formed term can be recognized by counting left and right parentheses and by checking the symbols adjacent to those parentheses (without referring to smaller terms). In more details, a string w is *balanced* if its length is at least 2, the number of left parentheses in w equals the number of right parentheses in w, and every proper initial segment of w contains more left parentheses than right parentheses. A string w is a *quasiterm* if w is a variable, or the (single-element) string `0`, or any string of the form `S(`w`)`, `+(`w`)` or $\cdot$`(`w`)` where `(`w`)` is a balanced string. A quasiterm t is a *term* if whenever `(`w`)` is a balanced substring of t, then either this substring is immediately preceded by the letter `S` and w is a quasiterm, or it is immediately preceded by `+` or $\cdot$ and w has the form u`,`v where u and v are quasiterms (notice that the typewriter font is used to indicate real symbols). With these definitions, one can show (inside PA) that terms have the expected properties. The remaining syntactic notions (*formula*, *free* and *bound* occurrences, *substitution* and *substitutability* of terms, and *proof*) can be treated similarly. However, two important functions are involved in the treatment: finding the number of occurrences of certain symbol in a given string, and the exponential function. Before dealing with formalized syntax, one has to show that the graphs of these functions are Δ_0.

To sum up, primitive recursion is a useful tool for proving recursiveness of some functions, and it is also used in some proofs in recursion theory. However, we do not need to have primitive recursion in the definition of partial recursive functions, and we do not need it when arithmetizing the logical syntax.

Basically all ideas in this talk are due to Pavel Pudlák, we just add some details. Pudlák also presented a proof (actually at least two different proofs) showing that the graph of the exponential function is Δ_0.

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Two Logics of Variable Essences and Descartes' Creation Doctrine

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Accounts of the relationship between essences and necessity are common in modal metaphysics, (Fine, 1995) being the most famous recent work. An interesting variation on this theme can be found in Descartes' views on the relations between true and immutable natures and eternal truths.[1] Descartes holds that essences determine a class of necessary truths (the eternal truths). Among the eternal truths are mathematical truths such as "the interior angles of a triangle sum to two right angles" and logical truths such as "contradictories cannot be true together". Among the mathematical truths, those setting out the foundations of his physical theory (governing the mechanistic interactions of bodies), are central to his scientific and philosophical system. However, he also holds that essences are freely created by God, and hence that God could have created them otherwise than he did. It seems to follow from this that Descartes is committed to the claims (1) that God eternal truths are necessary and (2) that God could have made the eternal truths false, which appear to contradict each other.

I think the correct reading of this view is bimodal – the necessity had by the eternal truths is not the modal dual of the possibility with which God could have made the eternal truths false. Call the former kind the "inner" or "i-" modalities ($\Box, \Diamond$), and the latter "outer" or "o-" modalities ($\blacksquare, \blacklozenge$). On my preferred reading, the i-modalities concern the essences which God did actually create, whereas the o-modalities concern the essences which God did not create, but could have created (hence, they concern God's own essence as whether or not God could have created a particular essence (or collection thereof) is a question of whether that essence is compatible with God's own essence). The informal truth conditions are as follows:

- $\Box\phi$ iff ϕ is made true by the essences of created things.
- $\Diamond\phi$ iff ϕ is not made false by ("is compatible with") the essences of created things.
- $\blacksquare\phi$ iff is made true by any collection of essences God could have created.
- $\blacklozenge\phi$ iff ϕ is not made false by all collections of essences God could have created.

This picture exploits a natural distinction between actual and non-actual essences. It is here motivated by appeal to Descartes' peculiar account of essence and modality, but there are many views in modal metaphysics which posit the existence of essences to explain certain modal facts, and in such a setting the relations between static and varying essences are of potential interest. The topic of this paper is to develop two logics of variable essences aimed to capture elements of the creation doctrine.

Both are bimodal logics, presented model-theoretically, both on the basis of a shared class of frames $\mathscr{F} = \langle W, N, R, S, D, e, \{X_i : i \in I\}_{|I| \leq \aleph_0} \rangle$ where

[1] Descartes' original work on the topic is mostly found in the correspondence (Cottingham et. al., 1991), but with short discussions in the replies to the Fifth and Sixth objections (Cottingham et. al., 1984)

- W is a set $\{\alpha, \beta, \gamma, \ldots\}$
- $N \subseteq W$ and $N \neq \varnothing$
- $R, S \subseteq W^2$
- D is a (countable) set $\{\overline{a}_1, \ldots, \overline{a}_n \ldots\}$
- $e : W \times D \longrightarrow \wp\wp(D)$
- $X_i : W \longrightarrow \wp(D)$

subject to the following constraints ($\&, \Rightarrow$ are (classical) metalanguage connectives):

(c1) R, S are equivalences, $R \subseteq S$

(c2) $\alpha \in N \;\&\; R\alpha\beta \Rightarrow \beta \in N$

(c3) $\alpha, \beta \in N \Rightarrow R\alpha\beta$

(c4) $X_i\alpha \in e(\alpha, \overline{a}), R\alpha\beta \Rightarrow \overline{a} \in X_i\beta$

(c5) $X_i\alpha \in e(\alpha, \overline{a}), R\alpha\beta \Rightarrow X_i\beta \in e(\beta, \overline{a})$

(c5') $\forall X_i, \overline{a}[X_i\alpha \in e(\alpha, \overline{a}) \iff X_i\beta \in e(\beta, \overline{a})] \iff R\alpha\beta$

R interprets the i-modalities and S the o-modalities with the usual truth conditions, and e assigns essences (the various X_i, which in the model interpret unary predicates) to objects at worlds. The use of the X_i's and e provide a simple approach to distinguishing essential from contingent predications. (c4), (c5) correspond to intuitive properties concerning this relation – respectively that any essential property is an i-necessary property, and that any essential property is i-necessarily essential. Furthermore, (c5') extends (c5) by requiring that i-modal facts co-vary with essential properties assigned to objects at worlds – cashing out the intuitive idea that facts concerning essences *determine* i-modal facts.

The first logic on this class of frames, that of *Classical Variable Essences* or **CVE**, is an extension of classical bimodal **S5**/U with the essence-interpreting mechanism sketched above. Worlds in the model are classical, and the differences between actual essences and merely possible essences are entirely cashed out in terms of a distinction between R-accessible and S-accessible worlds. An axiomatic system for **CVE** (with (c5) and not (c5')) is presented, soundness and completeness is proved, and the question for the system resulting from the addition of (c5') is discussed.

The second logic caters also to the additional commitment of Descartes' creation doctrine that *logic* is also under God's voluntary control. I discuss two natural ways of going about this - one, extending **FDE**, focuses on logical *truth*, and one, employing an *open worlds* construction, like that developed in Ch. 9 of (Priest, 2016), focuses on *consequence*. My focus is to adapt the mechanics of **CVE**, with the additional resources afforded by open worlds, to characterize the logic of non-classical variable essences, **NVE**.

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Quantum Logic Against Anti-Exceptionalism

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The evaluation of logical and metaphysical theories should follow the same criteria as those that are in place for the evaluation of scientific theories. This view is nowadays called "anti-exceptionalism" about logic and metaphysics. It upholds criteria like strength and simplicity, as well as consistency with the rest of our independently justified beliefs; for instance, it maintains that logical and metaphysical theories should not entail anything that collides with our scientific knowledge.

More particularly, anti-exceptionalism claims that our investigation of metaphysical modality should not be pursued "in isolation" from physical modality, which is sometimes justified in the following way: physical modality stands or falls with metaphysical modality, and it cannot be independently explained in terms of physical theories. Physical possibility can only be defined in terms of metaphysical compossibility with what in certain circumstances are the laws of nature (Williamson 2013, 2016).

What I argue in this paper is that physical modality is in fact independently explained in terms of physical theories. More specifically, I show that there are circumstances in which what is deemed physically possible according to a physical theory is not metaphysically compossible with what, in those circumstances, are the laws of nature. My argument is based on an analysis of the logical interpretation of quantum mechanics and proceeds in the following way.

Let a quantum calculus be a structure $\mathscr{Q} = \langle Q, \sim, \wedge, \vee \rangle$ that satisfies the usual conditions articulated in the logical interpretation, and let its semantics be given in terms of lattices, i.e., algebraic structures $\mathscr{L} = \langle L, ', \cap, \cup \rangle$ that also satisfy the usual conditions, including orthomodularity: a lattice $\mathscr{L}$ is orthomodular if and only if for any elements $a, b, c \in L$, $a \cup b \equiv a \cup (a' \cap (a \cup b)) = 1 \Rightarrow a \cup b = a \cup (a' \cap (a \cup b))$. It is weakly orthomodular if and only if for any elements $a, b, c \in L$, $a \cup b \equiv a \cup (a' \cap (a \cup b)) = 1 \Rightarrow (a \cup b) \cup c \equiv a \cup (a' \cap (a \cup b)) \cup c = 1$. In an orthomodular lattice, but not in a nonorthomodular, weakly orthomodular one, two elements are equivalent only if they are identical.

I start from the fact that there is no homomorphism between the quantum calculus, $\mathscr{Q}$, and its algebraic semantics, $\mathscr{L}$ (Paviĉić and Megill 1999, Paviĉić 2016). That is, quantum calculus can be interpreted by both orthomodular lattices and by nonorthomodular, weakly orthomdular ones. (It can be easily shown that the latter do not validate orthomodularity.) Thus, quantum logic is non-categorical. Then I argue that non-isomorphic lattice models of the calculus $\mathscr{Q}$ represent distinct physical possibilities. This is the basis for an independent account of physical modality, i.e., one that is not dependent on metaphysical modality. According to this account, both orthomodularity and weak orthomdularity describe physical possibilities.

Now consider an orthomodular lattice as a model of $\mathscr{Q}$. In this "possible world", orthomodularity is a law of nature. According to the anti-exceptionalist account of physical possibility, weak orthomodularity is physically possible, since its conjunction with the law of orthomodularity is true in any orthomodular world. This follows from the mathematical fact that an orthomodular latice is also weakly orthomodular. Next, consider a non-orthomodular, weakly orthomodular lattice as a model of $\mathscr{Q}$. In this other "possible world", weak orthomodularity is a law of nature. However, according to the anti-exceptionalist account of physical possibility, orthomodularity would be physically impossible, since its conjunction with the law of weak orthomodularity is false in all non-modular, weakly orthomodular worlds. This follows from the mathematical fact that a non-orthomodular, weakly orthomodular lattice cannot be orthomodular, as already pointed out above.

This proves that whereas on a metaphysically independent account of physical modality, based on a rigorous analysis of the logical interpretation of quantum mechanics, orthomodularity is physically possible, the anti-exceptionalist account of physical possibility entails that orthomodularity is physically impossible. Thus, our analysis shows that the above metaphysical explanation of physical modality collides with physical theory: the anti-exceptionalist account of physical possibility is inconsistent with the account of physical possibility independently explained in terms of physical theory.

My argument, however, does not entail that anti-exceptionalism, as a general view about metaphysics and logic, must be rejected. In particular, it does not entail that the idea that logical and metaphysical theories should not be developed in isolation from natural science is false. The argument shows only that the claim that physical modality stands or falls with metaphysical modality is false. If anti-exceptionalism is committed to this claim, than it has a problem.

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The Logic of Collective Acceptance

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In criticizing Philip Pettit's republican philosophy of popular control [4], Sean Ingham [2] introduces a distinction between two senses in which a given policy can be accepted by a group. According to Ingham's argument, Pettit confuses these two senses, using the stronger one in formulating general desiderata for popular control, but the other, weaker one in evaluating and defending a specific model of democratic decision-making.

Ingham explains his distinction using a formal, set-theoretic model. In line with social choice theory, he starts from a set X of potential policies, from which the government is supposed to select one policy within the boundaries set by popular control. Such popular control should in turn be a function of the norms endorsed by the citizens. Ingham models those norms in terms of the alternatives they rule out. That an agent i endorses a basic norm $Y \subseteq X$ means, hence, that i does not accept any alternative outside Y. An alternative is then acceptable relative to $\mathscr{N}_i \subseteq \wp(X)$, the set of norms endorsed by i, if and only if the alternative is permitted by every member of $\mathscr{N}_i$.[1]

Even with this relatively simple representation of norms and their relation to acceptance, there are at least two non-equivalent ways to specify the notion of group acceptance. On the first, an alternative is acceptable to a group iff it is acceptable in view of the set of all shared norms, where shared norms are norms endorsed by each agent in the group. Call this *shared norm acceptability*. On the second specification, which Ingham refers to as *universal acceptability*, an alternative is acceptable for the group if and only if it is accepted by each member of the group, in view of their respective individual norms.

It is tempting to link this distinction to other notions in the theory of democratic decision-making, such as Rawls' *overlapping consensus* [5], Sunstein's *incompletely theorized agreements* [6], and the notion of *meta-agreement* [1, 3]. One should however be careful not read too much into the formalism just presented. After all, a given citizen may reject a certain subset of the alternatives for various reasons of a very different nature, and one cannot simply reduce such reasons to subsets of alternatives. In particular, the mentioned concepts often crucially refer to the general principles and values that are used to argue for or against accepting a certain alternative, which are not represented in Ingham's formalism.

So rather than an adequate representation of these intricate and rich concepts, we will argue that Ingham's distinction and model provide a useful abstraction to think about such forms of group acceptance in exact terms, and show how it allows us to tackle some interesting questions. What exactly is implied by shared norm acceptability, what does universal acceptance entail? How do these various notions behave and interact, when relativized to different coalitions? Under what conditions do these notions coincide? What is their logic, and how does it interact with statements concerning the norms a given agent endorses? How much does Ingham's distinction, and the logic of these notions more generally, depend on the logical properties of norms and the relation between norms and acceptability?

In this paper, we present a logic that allows us to study these questions in exact terms. Starting from a finite set of agents $N = \{i_1, \ldots, i_n\}$, we introduce a formal language that

[1] Cf. [2, p. 106]: "[i]f an option x is permitted by every policy-making norm that she [= the agent in question] accepts, then I will say that she finds it acceptable; if it violates a policy-making norm she accepts, I will say that she finds it unacceptable."

features three types of modal operators, beside classical propositional logic and a universal modality: (i) $\Box_i$, to express norms that are endorsed by the agent i; (ii) $\Box^s_G$, to express properties of alternatives that conform to all shared norms among group G; and (iii) $\Box^u_G$, to express properties of alternatives that are acceptable to all members of G.

This formal language is interpreted using neighbourhood models of the type $\langle W, \langle \mathscr{N}_i \rangle_{i \in N}, V \rangle$, where W is a non-empty set of worlds, each $\mathscr{N}_i$ is the neighbourhood function that represents the norms endorsed by a given agent at each world $w \in W$, and V is a valuation function. The semantic clauses for the three above operators run as follows, where $w \in W$:

1. $M, w \models \Box_i \varphi$ iff $\|\varphi\|^M \in \mathscr{N}_i(w)$
2. $M, w \models \Box^u_G \varphi$ iff $\bigcap_{i \in G} \bigcap \mathscr{N}_i(w) \subseteq \|\varphi\|^M$
3. $M, w \models \Box^s_G \varphi$ iff $\bigcap \bigcap_{i \in G} \mathscr{N}_i(w) \subseteq \|\varphi\|^M$

We will discuss a number of salient properties of this logic and propose an axiomatization for it. Our completeness proof builds on the logics for pointwise intersection from [7], with some non-trivial adjustments. After that, we will consider various frame conditions and corresponding axioms, focussing in particular on conditions that express certain types of agreement among agents, and the impact they have on the two notions of group acceptance and their interrelation. As we will show, for models where the neighbourhood functions $\mathscr{N}_i$ are monotonic, shared norm acceptability is analogous to shared belief in multi-agent doxastic logic, whereas universal acceptability is analogous to distributed belief. Under this alternative interpretation, our logic can be seen as grounding both notions (and individual belief) in evidence as represented by the neighbourhood functions.

In the final part of the talk, we will consider several enrichments and variations of the simple framework presented, along the following dimensions: (i) relativizing the space of alternatives X to the world of evaluation, thus allowing for a more natural interpretation of the semantics in terms of forward-looking acceptance; (ii) further relativizing X to the agents, thus encoding epistemic/doxastic aspects of individual and collective acceptance; (iii) representing reasons for norms explicitly in the object language and semantics, in order to give a more adequate model of the above-mentioned notion of meta-agreement; (iv) replacing Ingham's notion of acceptance with one that is conflict-tolerant, arguably leading to a breakdown of the implication from universal acceptance to shared-norm acceptance.

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