LOGICA 2023

ABSTRACTS

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Type-theoretic expressivism

DAVID CORFIELD
University of Kent, UK
e-mail: d.corfield@kent.ac.uk

The thesis of *logical* expressivism, as proposed by Robert Brandom (2000) and others, is that the role of logical vocabulary is to allow the explicit articulation of inferential relations. This occurs when we endorse instances of reasoning by means of assertions employing logical vocabulary, rather than merely implicitly doing so by reasoning in certain ways. For instance, in the case of propositional logic, implication is understood as allowing the explicit endorsement of the inference of one proposition from another via assertion of the associated hypothetical proposition.

Now from the perspective of dependent type theories, such as homotopy type theory (Corfield 2020), propositional and first-order logic may be considered as mere fragments by restrictions on the dependency structure and on the kinds of types allowed in its judgements. In this talk I will be exploring the extent to which the vocabulary of homotopy type theory may also be given an expressivist reading. I will also touch on modal variants.

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A new semantics for conditionals

WESLEY H. HOLLIDAY
University of California, Berkeley, USA
e-mail: wesholliday@berkeley.edu

Conditionals in their different flavors—material, strict, indicative, counterfactual, probabilistic, constructive, quantum, etc.—have long been of central interest in philosophical logic (see, e.g., Lewis 1912, Stalnaker 1968, Lewis 1973, Adams 1975, Edgington 1995, Nute and Cross 2001, Egré and Rott 2021). In this talk, we will discuss a new semantics for conditionals introduced in recent work on the representation of lattices with conditional operations (Holliday 2023, § 6). We define a *preconditional* on a bounded lattice to be a binary operation satisfying the following axioms:

1. $1 \to a \leq a$;
2. $a \wedge b \leq a \to b$;
3. $a \to b \leq a \to (a \wedge b)$;
4. $a \to (b \wedge c) \leq a \to b$;
5. $a \to ((a \wedge b) \to c) \leq (a \wedge b) \to c$.

Familiar examples of bounded lattices equipped with a preconditional include Heyting algebras, ortholattices with the Sasaki hook, and Lewis-Stalnaker-style conditional algebras satisfying the so-called Flattening axiom (Mandelkern Forthcoming). We have shown that every bounded lattice equipped with a preconditional can be represented using a relational structure $(X, \lhd)$ (suitably topologized), yielding a single relational semantics for conditional logics normally treated by different semantics, as well as generalizing beyond those semantics. We will discuss further developments in this approach to conditionals.

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Hume's law and other barriers to entailment

GILLIAN RUSSELL
Dianoia Research Institute at Australian Catholic University, Melbourne, Australia
e-mail: gillian.russell@acu.edu.au

This paper looks at how to prove Hume's law as one of a number of interrelated barriers to entailment and in particular what to make of proposed counterexamples that require complex logics, such as those based on Ought implies Can.

Logic, feminism, and feminist logic

SARA L. UCKELMAN
Durham University, UK
e-mail: s.l.uckelman@durham.ac.uk

There has been a long history of tension between feminists and feminist philosophy, on the one hand, and logic, on the other hand. This tension expresses itself in many ways, including claims that logic is a tool of the patriarchy, that logic/rationality/analytical tools in philosophy need to be rejected if women are to fully participate, that women = body and man = mind, that to do feminist philosophy one must do it as a situated, embodied person, not as an impersonal, disembodied mind, that logic is "a masculine subject". However the tension is expressed, it is women in logic and women logicians who are caught in between. The goal of my paper is to explore a conception of logic that not only is not inconsistent with being a feminist, but is actively welcoming of women as logicians.

Contributed talks

Meaning and identity of proofs in a bilateralist setting: A two-sorted typed lambda-calculus for proofs and refutations

SARA AYHAN
Ruhr University Bochum, Germany
e-mail: sara.ayhan@rub.de

In this talk we will be concerned with questions about sense, reference, identity and synonymy of proofs and refutations in a setting of bilateralist proof-theoretic semantics. Proof-theoretic semantics is an approach claiming that the meaning of logical connectives is not given by model-theoretic concepts, like truth tables etc., but rather by the rules governing their use in derivations. Traditionally, for that purpose the *proof* conditions are considered (see, e.g., Dummett, 1991; Prawitz, 1973). There is, however, also the opposing position (see, e.g., López-Escobar, 1972) claiming that it is rather the *refutation* conditions which ought to be taken into account for such an approach. A bilateralist conception of proof-theoretic semantics (see, e.g., Rumfitt, 2000) aims to combine these two accounts in saying that verification conditions and falsification conditions should be taken on a par: both should be considered equally in determining the meaning of the logical connectives.

In this context there are two issues that will be addressed in this talk. Firstly, we want to give an account on how it is possible to distinguish between sense and reference of derivations in a spirit of proof-theoretic semantics. It is generally assumed that there can be different linguistic representations of the *same underlying proof*. In these cases it could be said - building upon Frege's (1892) famous distinction - that they differ in sense but not in denotation. Thus, the question of when two derivations are identical seems reasonable and has been dealt with extensively in the literature (see, e.g., Kreisel, 1971; Martin-Löf, 1975; Prawitz, 1971). This question concerns only the notion of *denotation*, though, the other question, namely what exactly could be thought of as the *sense* of a derivation has not been considered in the same way. Thus, in this talk we will give a concrete conception of what constitutes the sense of derivations and thereby we will be able to make more fine-grained distinctions: not only can we give an answer to the question of when two derivations are identical but also to the question of when they must be considered *synonymous*.

Our second aim is concerned with the bilateralist perspective and the relation between proofs and refutations in the context of such a Fregean distinction. Therefore, we will introduce a type theory for the bi-intuitionistic logic, `2Int`, by Wansing (2016; 2017), i.e., a logic conservatively extending intuitionistic logic with a dual connective to implication, called 'co-implication'. Thus, we will use the Curry-Howard correspondence, which has been well-established between the simply typed λ-calculus and natural deduction systems for intuitionistic logic, and apply it to a bilateralist proof system displaying two derivability relations, one for proving and one for refuting. The basis will be the natural deduction system `N2Int`, which we will turn into a term-annotated form. Therefore, we need a type theory that extends to a two-sorted typed λ-calculus displaying two polarities in the terms. Using our account about what constitutes sense and reference of derivations, we will argue, then, that in a system with rules characterizing both proof and refutation conditions of the connectives certain proofs and refutations can be seen as different ways of representing the *same object*, i.e., they would only differ in sense but not in denotation. More specifically, we will define a duality function which allows us to identify every proof (resp. refutation) of a formula in our system with a refutation (resp. proof) of the 'dual formula'. This identification is motivated by showing that the underlying constructions of the derivations are essentially the same. Thus, such a view would yield the (from a bilateralist point of view) desired balance between proofs and refutations: they are considered as equal; neither concept is reduced to the other and no preference is given to one or the other. However, this does not lead to a complete collapse between proofs and refutations (and thus, one might object, to a return to unilateralism), since their *senses* must be clearly distinguished: although there is *one* derivational construction, this is presented in very different ways, one by proving something and the other by refuting something.

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Free logic admitting truth-value gaps and gluts

Libor Běhounek
University of Ostrava, Czech Republic
e-mail: libor.behounek@osu.cz

Antonín Dvořák
University of Ostrava, Czech Republic
e-mail: antonin.dvorak@osu.cz

In many settings, there is a need to quantify over non-denoting terms—e.g., when dealing with definite descriptions or partial functions; in logical analysis of fictional discourse; and in predicate modal logic with variable domains, where some individuals may not exist in all possible worlds. Quantification that admits non-denoting terms is the domain of *free logic,* which is commonly developed in several variants [2]: *negative,* where all atomic formulas containing empty terms are considered false; *neutral,* where all atoms with empty terms (except for atoms of the form "t exists") are considered truth-valueless; and *positive,* which admits truth as well as falsity and truth-value gaps for formulas containing empty terms. Of these alternatives, positive free logic is arguably the most flexible one, as it admits such intuitive assumptions as the truth of the statement $Pegasus = Pegasus$, the falsity of the statement $Pegasus = Zeus$, and the truth-valuelessness of the statement $1/0 = -1/0$. A convenient form of positive free logic is based on the *dual-domain semantics* [6], which distinguishes the *inner domain* D_1 of existing individuals and the *outer domain* $D_0 \supseteq D_1$ that includes fictive denotations of non-denoting terms. Within the outer domain, the inner domain is delimited by the *existence predicate* (E!); thus, $\mathrm{E!}\,t$ is true iff $\|t\| \in D_1$. Dual-domain positive free logic incorporates the ordinary *inner quantifiers* ($\forall, \exists$) that range over the inner domain of existing individuals and, additionally, the *outer quantifiers* (Π, Σ), which make it possible to formalize such statements as "some things do not exist". The truth-valueless statements of positive free logic require using a suitable background logic that admits truth-value gaps, for instance the three-valued strong Kleene logic K_3. In the framework of dual-domain free logic, the inner and outer quantifiers and the existence predicate provide the expressive means to make the existence assumptions of inferences with non-denoting terms explicit.

In this contribution we aim to outline and discuss an extension of dual-domain positive free logic to its variant admitting contradictory objects in the outer domain. There is a plethora of reasons to accommodate contradictory nonexistent objects, as they include objects defined by contradictory descriptions (a round square both is and is not round) and instances of various paradoxical concepts in philosophy (e.g., the paradox of the stone under omnipotence), mathematics (the notion of infinitesimal), and fiction (inconsistent depictions in stories). A philosophical treatment of inconsistent nonexistent objects can also be found in Meinong [5].

Positive free logic admitting inconsistent objects needs to work with truth-value gluts in addition to truth-value gaps. To avoid trivialization by the inconsistencies, its underlying logic needs to be paraconsistent, and ideally *relevant,* as it is typically undesirable to draw irrelevant conclusions from conflicting assumptions about nonexistent objects. For the sake of generality and simplicity, for the underlying logic we choose a suitable predicate extension of the Dunn–Belnap logic FDE [7], which has a simple four-valued semantics (with the truth-values *true, false, neither,* and *both*) and is a fragment of prominent relevance logics. Dual-domain free logic can be defined over FDE similarly as has been done by Carnielli and Antunes in a similar setting [3]. (Carnielli and Antunes employ a three-valued logic with a glut, so they also admit contradictory objects; however, their free logic does not admit truth-value gaps.)

The expressive and inferential weakness of the logic FDE poses several challenges for developing free logic upon its basis, which necessitate applying some tricks. For instance, the expressibility of certain claims on existence and non-existence requires considering separate primitive quantifiers over the inner domain D_1 and over its complement $D_0 \setminus D_1$ (cf. [8]). Alternatively, to ensure the expressibility (and axiomatizability) of the claim that existing objects are consistent and governed by two-valued logic, it may be desirable to expand the language of FDE by the consistency connective. Since we only aspire to develop a theory of free quantification, rather than to solve Russell-style paradoxes, the addition of the consistency connective can be justified despite its non-paraconsistent nature.

A major flaw in the outlined three- and four-valued free logics, however, is their truth-functionality, which results in lottery-type paradoxes: in many scenarios, it is reasonable to assume the truth (i.e., the truth value *true*) of a disjunction, but the truth-valuelessness (i.e., the truth value *neither*) of all the disjuncts; however, in these cases, the disjunction is evaluated as truth-valueless (i.e., *neither*) in FDE (or K_3). Consequently, the outlined truth-functional four-valued free logic can only be regarded as a toy model of free quantification over inconsistent objects. The flaw can be remedied by considering a four-valued unary modality $\mathcal{T}$ (or a pair of two-valued modalities $\mathcal{T}^+, \mathcal{T}^-$), representing Belnap's interpretation [1] of the four truth values in terms of the proposition being "told true" (or not) and "told false" (or not); it is sufficient to assume a two-layered syntax and simple second-order semantics for this modality (similar to [4]). However, we will only hint at this refinement of the four-valued free logic in the talk and will primarily focus on the definition, motivation, design choices, and basic properties of the four-valued dual-domain positive free logic in its simpler, truth-functional form.

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A dynamic epistemic logic of attention with attention change and limits on attentional capacities

GAIA BELARDINELLI
University of Copenhagen, Denmark
e-mail: belardinelli@hum.ku.dk

THOMAS BOLANDER
Technical University of Denmark, Denmark
e-mail: tobo@dtu.dk

Background: Logics of announcements One way of learning from experience is to update our beliefs based on receiving stimuli from the outside world. One of the most basic logics for modelling this in a multi-agent setting is public announcement logic (PAL) (Plaza, 1989): An agent or the environment is announcing or revealing some formula φ, and all agents update their beliefs with φ. PAL can be generalised to dynamic epistemic logic (DEL) (Baltag et al., 1998) where the actions can be much more complex (modelled by so-called event models). Most of the existing generalisations of PAL go in the direction of restricting who pays attention to the announcements (like private announcements) or restricting what can be announced. Here we are instead interested in generalising PAL by restricting *which parts* of an announcement is payed attention to. This might at first sound strange, since it seems that either we pay attention to an announcement or we don't. However, consider a situation where the "announcement" is exposure to some visual stimuli, for instance looking at a picture or watching a video. After having watched the video, we might not be able to tell the color of the shirt of a person walking in the background. We might simply not have *payed attention*.

A recent logic of attention In very recent work (Belardinelli at al., 2023), the authors use DEL to create models of agents who might only pay attention to a subset of what is being announced/revealed. The static models are Kripke models over a language with additional atoms $\mathsf{h}_a p$ expressing that "agent a pays attention to p".[1] To define the dynamics, they then construct a "stimuli exposure event model" that expresses what each agent learns when being exposed to stimuli represented by a conjunctive formula $\ell(p_1) \wedge \cdots \wedge \ell(p_n)$ where $\ell(p_i) = p_i$ or $\ell(p_i) = \neg p_i$ for all i (a conjunction of literals). These are called *event models for propositional attention*, and generalise work of Bolander et al. (2016). In our video example, a literal $\ell(p_i)$ could express that "the person walking in the background is wearing white" or "the tree in the upper right is an oak", and then watching the video means getting exposed to the conjunction of all these literals. The $\mathsf{h}_a p$ atoms control which of these literals we actually learn, as well as what we learn about what other agents learn. The event model is complex, but can be compactly described in terms of two simple principles:

ATTENTIVENESS If in the actual event agent a pays attention to p, then, in any event a considers possible, $\ell(p)$ is revealed and a pays attention to p.[2]

INERTIA If in the actual event agent a does not pay attention to p, then, in any event a considers possible, nothing about p is revealed.[3]

[1] In this submission, we assume *attention introspection*: if $w \in V(\mathsf{h}_a p)$ and $(w, v) \in R_a$ then $v \in V(\mathsf{h}_a p)$.

[2] Formally, this becomes a constraint on the accessibility relation Q_a of agent a expressing that $(e, f) \in Q_a$ only if $pre(e) \models \mathsf{h}_a p$ implies $pre(f) \models \ell(p) \wedge \mathsf{h}_a p$.

[3] The added constraint that $(e, f) \in Q_a$ only if $pre(e) \not\models \mathsf{h}_a p$ implies $pre(f) \not\models p$ and $pre(f) \not\models \neg p$.

ATTENTIVENESS expresses that agents who pay attention learn what they pay attention to *and* they become aware that they are paying attention to it. INERTIA expresses that agents who don't pay attention to something preserve their previous beliefs about it.

Our first contribution: Attention change Human attention is often described as a spotlight that highlights parts of the environment that agents focus on and learn. To allow new learning, attention must move from content to content, dynamically re-focusing on new information. In this work, we extend the above framework to account for change in attention focus. There are two different ways in which agents might change what they attend to: (1) They might decide to start paying attention to some p, for instance if it is relevant to their current goals. This is *top-down attention*, also called goal-oriented, voluntary or directed attention (Bridewell et al., 2016). (2) They might get exposed to some stimulus p that inevitably attracts their attention, for instance when p represents a sudden loud noise happening nearby, or ads popping up during a video we are looking at. This is often referred to as *bottom-up attention*, also called stimulus-driven, involuntary or captured attention (Bridewell et al., 2016). Below we describe our proposed models for top-down and bottom-up attention change.

Top-down Top-down attention change is modelled using event models with postconditions (van Ditmarsch et al., 2016). Inspired by Bolander et al. (2016), we consider assignment expressions $+\mathsf{h}_a p$ meaning that agent a decides to pay attention to p.[4] The event model representing the assignment $+\mathsf{h}_a p$ contains two events, one actual event e_1 in which the postcondition specifies that agent a starts paying attention to p, and another event e_2 representing that nothing happens. We assume that all agents except a have an edge from e_1 to e_2, meaning that we are here modelling *private* attention change: Agents cannot directly observe when other agents change their attention, i.e., they preserve uncertainty regarding the attention of others.[5]

Bottom-up Bottom-up attention is modelled via event models for propositional attention, extended with postconditions (as above) and parametrised by a set G of *attention-grabbing literals*. The literals in G are a subset of literals from the revealed formula $\ell(p_1) \wedge \cdots \wedge \ell(p_n)$ that represent the part of the announcement that inevitably attracts all agents' attention. These events are *public* attention-change events where, besides learning that their own attention changed, agents also learn that the other agents' attention changed. The bottom-up attention dynamics are given by ATTENTIVENESS together with the following conditions:

BOTTOM-UP ATTENTIVENESS If a revealed $\ell(p)$ is attention-grabbing, then in any event that a considers possible, $\ell(p)$ is revealed and all agents come to pay attention to p.[6]

INERTIA* If in the actual event agent a does not pay attention to p and $\ell(p)$ is not attention-grabbing, then, in any event a considers possible, nothing about p is revealed.[7]

Our second contribution: Attentional capacities A crucial aspect of human attention is its limited capacity, the fact that we usually can't pay attention to everything. So even if all truths about the world were revealed to us in one instant, we wouldn't be able to process them all. We extend our model by introducing limitations on the attentional capacities of agents, taking a very basic approach with a fixed bound $m \in \mathbb{N}$ on how many atoms each agent can pay attention to. No matter whether an attention change is top-down or bottom-up, it has to obey

[4] And, symmetrically, assignment expressions $-\mathsf{h}_a p$ meaning that agent a stops to pay attention to p.

[5] The event model is as follows, using notational conventions from Bolander et al. (2011): $\underline{e_1 : \langle \top, \mathsf{h}_a p \rangle} \xrightarrow{\; b \in Ag \setminus \{a\} \;} e_2 : \langle \top, \top \rangle$, where Ag is the set of agents. Applying several such updates in sequence then allow us to model top-down attention change for multiple propositional atoms and/or agents.

[6] The added constraint that $(e, f) \in Q_a$ only if $p \in G$ implies $pre(f) \models \ell(p)$ and $post(f) \models \bigwedge_{p \in Ag} \mathsf{h}_a p$, where postconditions are modelled as conjunctions of literals (Bolander et al., 2011).

[7] This is as INERTIA except we add $\ell(p) \notin G$ to the antecedent of the condition.

the capacity bounds, i.e., if the agent is already paying attention to m atoms, she can't come to pay attention to more. We achieve this by adding conjuncts to our event preconditions, for instance adding to the event e_1 above that a is currently paying attention to less than m atoms.[8]

Additional contributions In the oral presentation (and full paper), we will also consider axiomatization of the logic (revising the axiomatization of Belardinelli at al. (2023)), provide examples of it's use in modelling inattentional blindness (Mack et al., 1998), discuss potential applications and relate to the philosophical literature on attention, in particular Watzl (2017).

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[8] This is not too difficult. Note that we can express that agent a is paying attention to fewer than m atoms by the formula $\bigwedge_{\{p_1,\dots,p_m\}\subseteq P, p_i \neq p_j \text{ for all } i\neq j} \neg(\mathsf{h}_a p_1 \wedge \cdots \wedge \mathsf{h}_a p_m)$ where the set of propositional atoms P is assumed finite.

Disquotation, minimality and the logical force of the truth predicate

PIETRO BROCCI
Scuola Normale Superiore di Pisa/Classe di lettere e filosofia, Italy
e-mail: pietro.brocci@sns.it

The popularization of the deflationist doctrine on truth has brought new attention to disquotational principles, i.e. $T(A) \leftrightarrow A$. The reason for this is that, according to deflationists, truth is merely a logico-mathematical device and it should be logically represented by principles that express its function. Since disquotational principles are generally successful in doing so, disquotational theories of truth are being investigated now more than ever. In particular, we are interested in the formulation of *axiomatic* theories of disquotational truth and how these formal frameworks allow us to measure the proof-theoretic power of truth principles over a certain base theory. This leads to the question of how much power can be achieved by disquotational principles alone. Let us stress, that we are not interested in the *absolute* strength a theory can achieve, but rather in the comparison of the mathematical power of different versions of disquotational axioms for truth over the same base theory.

Which disquotational principles should we accept? The liar paradox prevents us from naively accepting all instances of disquotational principles. One way to solve this is to consider only disquotational principles for sentences where the truth predicate does not occur (typed theories). However, these theories are conservative over the base theory and therefore mathematically uninteresting. Thus, we focus our attention on type-free (or self-referential) theories of truth, where truth can be predicated of sentences containing the truth predicate. One might be tempted to consider maximal consistent sets of disquotational principles; however, there are uncountably many of them and they are not recursively enumerable (5). Therefore, we observe that it is by no means a trivial task to single out consistent and proof-theoretically interesting axiomatic theories of disquotational truth.

For this reason, in the first part of the paper, we assess the status and logical force of type-free axiomatic theories of disquotational truth in the literature, in particular the ones formulated in classical logic. We survey the theories formulated by Halbach (4), Schindler (7) and Picollo (6). Picollo banishes all the disquotational principles that involve unfounded sentences; at first, a proof-theoretic evaluation reveal that these disquotational principles are proof-theoretically weak over Peano Arithmetic. A relevant proof-theoretic power for the theory is then achieved by changing the base theory, we argue that this move excludes the possibility of comparing this theory with the others (since Peano arithmetic is the underlying framework of most theories of truth). Schindler's proposal achieves the strongest proof-theoretic power by translating the language of second-order arithmetic into the one of first-order truth (this relationship was already shown in (2)). This suggests a strategy to formulate mathematically interesting disquotational theories of truth.

We turn to Halbach's theory, PUTB, which allows only disquotational principles that involve T-positive sentences, i.e. sentences where the truth predicate occurs under the scope of an even number of negation symbols. It is folklore (see (2) and (4)) that PUTB is equivalent to the axiomatization of Kripke's fixed point theory of truth, i.e. the Kripke-Feferman (KF) theory; moreover, from a result in (2) it follows that PUTB is proof-theoretically as strong as the theory that states the existence of fixed-points for arbitrary positive inductive definitions, $\widehat{\mathsf{ID}}_1$. We conclude this part by observing that the strategy used by Schindler can lead us to consider whether Halbach's theory can be made stronger.

The second part of the paper contains the main results. The goal is to formulate a new disquotational theory of T-positive truth which is stronger than Halbach's. Motivated by Burgess (1) minimal version of KF that captures the *minimal* fixed-point of Kripke's construction, we formulate a minimal version of PUTB as well. To do so, we extend it by means of a minimality principle, the idea behind this is to truth-theoretically capture the content of the theory ID_1, i.e. the theory that states the existence of *minimal* fixed-points for arbitrary positive inductive definitions. Then, we prove that this new theory, PUTB_μ, is proof-theoretically as strong as ID_1. This makes PUTB_μ as strong as Burgess' theory, KF_μ.

We have (partially) replicated the equivalence between the axiomatization of Kripke's fixed point truth and axiomatic T-positive disquotational truth at the level of the *minimal* fixed point (i.e. at the impredicative level). However, our results and Burgess' are not sharp, it remains to prove that ID_1 can prove KF_μ and PUTB_μ.

These results of proof-theoretical equivalence establish that two different conceptions of truth, over the same base theory, have the same arithmetical consequences. That is, they prove the same truth-free theorems. It can be argued that this is not enough to show that these concept of truth are compatible. For this purpose, Fujimoto in (3) introduces the notion of relative truth definability, which is a fine-grained tool that can be used to establish a conceptually more meaningful relationship between truth theories. Therefore, we conclude by proving that KF_μ is relative truth definable in PUTB_μ.

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Carnap's problem, definability and compositionality

PEDRO DEL VALLE-INCLAN
Scuola Normale Superiore, Pisa, Italy
e-mail: pedro.delvalleinclan@sns.it

According to inferentialists the meaning of logical vocabulary is determined by its use in inferences. The hard-line versions of the position developed by Došen (1989) or (Peregrin 2014) characterise meaning directly in terms of inference rules. Being a certain connective, on these accounts, just *is* being governed by certain rules. A moderate form of inferentialism is favoured by Hacking (1979), Boghossian (2000) or Murzi and Topey (2021). According to moderate inferentialists we can (in some sense) 'read' the meaning of logical words off their role in inferences, even though meanings and inferential roles should not be equated.

A well-known result by Carnap poses a problem for moderate inferentialism. In his *Formalization of Logic* (1943), Carnap pointed out that there are non-normal interpretations of classical logic: non-standard interpretations of the connectives and quantifiers that are nevertheless consistent with the classical consequence relation of the appropriate language. So, if we take inferential roles to be given by consequence relations, the meaning of classical logical vocabulary cannot be read off inferential roles. Let us call this Carnap's Problem.

In a recent paper Bonnay and Westerståhl (2016) put forward a solution to Carnap's Problem. Their approach is to limit the space of possible interpretations by 'universal semantic constraints'. According to Bonnay and Westerståhl, if we restrict attention to interpretations that are (a) compositional, (b) non-trivial and (c) in the case of the quantifiers, invariant under permutations of the domain, Carnap's Problem is avoided.

In this talk I will show that Bonnay and Westerståhl's approach does not work, and argue that the reasons behind the failure of their proposal reveal two major obstacles for a solution to Carnap's Problem in first-order settings.

The first problem with Bonnay and Westerståhl's proposal concerns the main result of their paper:

(BW) Let $\mathcal{L}$ be a language with $\forall$ primitive, let $\mathcal{M} = (D, I)$ be an $\mathcal{L}$-structure and let $Q \subseteq \mathcal{P}(\mathcal{D})$ be the denotation of $\forall$ (seen as a generalized quantifier). Then a weak model $\mathcal{M}, Q$ is consistent with the classical consequence relation for $\mathcal{L}$ iff Q is a principal filter closed under the interpretation of terms in $\mathcal{M}$.

Crucially, they only prove (BW) for first-order languages supplemented with predicate *variables* (in other words, for *second*-order languages without second-order quantifiers). I will show —adapting the methods of (Antonelli 2013)— that (BW) fails for first-order languages, and as a result the normal interpretation of quantifiers is not fixed. The underlying problem here regards definability. It is well-known that given a first-order language and a structure for it, there usually are subsets of the domain that cannot be defined by a formula. This general fact can be easily exploited to define non-normal interpretations, and makes Carnap's Problem for first-order languages particularly challenging.

The second problem with Bonnay and Westerståhl's approach is the way they define normal interpretations. In the first-order case, their normal interpretations either violate compositionality or beg the question against non-normal interpretations of the connectives. I will show that, once we redefine normal interpretations to avoid this problem, compositionality, non-triviality and invariance under permutation don't pin down the standard interpretation of classical logical vocabulary. In this case the underlying issue concerns compositionality. The usual semantics for first-order logic is *not* compositional. While there are ways of giving compositional semantics for first-order languages, they all involve accepting a vast array of possible semantic values. This, in its turn, makes Carnap's Problem all the more difficult to solve, since a richer set of semantic values entails a richer set of non-normal interpretations to choose from. Thus, while demanding compositionality is *in general* helpful to rule out deviant interpretations, in the first-order case it is counter-productive.

Finally, Bonnay and Westerståhl's use of predicate variables is somewhat analogous to the appeal to 'open-endedness' in other attempts at solving Carnap's Problem (e.g. McGee 2000, Murzi and Topey 2021). In the final part of the talk I will discuss how the present results relate to those approaches, and explore some ways to circumvent the problems they give rise to.

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Syntactic β-conversion, partiality, and dual procedures

MARIE DUŽÍ
VŠB-Technical University of Ostrava/Department of Computer Science, Czech Republic
e-mail: marie.duzi@vsb.cz

BJØRN JESPERSEN
University of Groningen/Faculty of Philosophy, The Netherlands
e-mail: b.t.f.jespersen@rug.nl

Transparent Intensional Logic (TIL) is a higher-order, hyperintensional λ-calculus with a procedural semantics. The terms of the ideography of TIL denote procedures producing mappings rather than the mappings themselves. Mappings are semantically secondary, while the procedures are semantically primary. Procedures are *sui generis* objects of the ontology of TIL. They include a pair of dual procedures operating on lower-order procedures, namely *Trivialization*, ${}^0C$, and *Double Execution*, ${}^2C$. While ${}^0C$ *displays* the procedure C as an object to operate on, ${}^2C$ *cancels* the displaying effect, as ${}^2C$ produces what is produced (if anything) by the procedure produced by C. It means that while in ${}^0C$ the procedure C does not occur in the *execution* mode, in ${}^2C$ it does.

When building up a TIL deduction system, the definition of correct substitution is fundamental. The correctness of λ-conversions, in particular β-conversion, is based on this definition. The validity of these conversions is a necessary condition for the proper use of the so-called *substitution method* that operates on procedures occurring within *hyperintensional contexts*. The basic principle of substitution is that one can substitute only for variables occurring free. Hence, the definition of free occurrence of a variable is also fundamental. Tichý, the founder of TIL, defined correct substitution in a restrictive way in (1988). He was careful not to substitute into a context within the scope of a Trivialization, where Trivialization makes a context hyperintensional. For Tichý, as soon as a variable occurs within the scope of a Trivialization, it is *Trivialization-bound*, hence displayed, and thus not free for substitution.[1] Tichý proved the basic theorem covering the validity of substitution and thus also the validity of λ-conversions, namely the Compensation Principle, thereby proving consistency. Yet, Tichý proved this principle only for procedures of order 1, thus ignoring the hyperintensional levels of TIL. Therefore, he did not get around to considering the fact that Double Execution cancels the effect of Trivialization.

For this reason, Duží et al. (2010) aimed to extend the definitions of free variable and correct substitution to also include those occurrences of variables that, apparently, become free due to the duality of Double Execution and Trivialisation. As Double Execution cancels the effect of Trivialization, a variable occurring within the scope of a Trivialization can become free for substitution, if it occurs also within the scope of Double Execution. Following (2010), the definition of substitution, as well as its correctness, seemed to be a completed task. TIL has been applied not least to natural-language processing, and when analyzing natural-language sentences, everything seemed to be all right.

However, Kosterec (2020), (2021) demonstrated inconsistencies in TIL that are due to too liberal a definition of a variable occurring free for substitution, which in turn goes hand in hand with too liberal a definition of the execution mode for occurrences of procedures. Some conversions that were apparently proved to be equivalent transformations turned out not to be

[1] Tichý did not use the terms 'displayed' and 'executed'. These terms were introduced in Duží et al. (2010) to distinguish between the two basic modes in which a TIL procedure can occur.

equivalent, as Kosterec discovered a similar mistake in the proof of the Compensation Principle in (2010). There are limiting cases where the redux procedure does not v-produce (i.e., produce relative to a valuation function v) the same entity as does the contractum procedure. However, Kosterec did not propose a solution. In (2020) he proposed a new version of the basic definitions, but it turned out that new inconsistencies cropped up, and so a proof of consistency is still outstanding. The goal of the present paper is to work toward filling this gap.

Our diagnosis is that the quintessence of the problem stems from running two levels of abstraction together: the 'syntactic' level (i.e., the sheer structure of a procedure), and the semantic level of evaluation of procedures (i.e., the level of computing what is produced by a given procedure with respect to a valuation of free variables occurring in the procedure).[2] Pezlar (2019) speaks about two notions of computation in TIL. One is syntactic computation, which corresponds to term rewriting in ordinary λ-calculi, and the other is semantic computation, which might be compared to *term interpretation*. Syntactic computation is specified by the rules of λ-conversion. These are α-, β-, and η-conversion. The most problematic of them is β-conversion, because in TIL we work with *partial* functions, and it has been proved that β-conversion by name is not an equivalent transformation in the logic of partial functions.[3] In a simple and general form, this conversion can be specified like this: $[\lambda x F(x) A] \Rightarrow F(A/x)$, where $F(A/x)$ arises from F by a correct substitution of A for the variable x. Hence, the fundamental issue is to define correctly the occurrence of a variable free for substitution. Bear in mind that, in TIL, variables are procedures. Since this definition is closely connected to the distinction between a procedure occurring either displayed or executed, this definition also calls for repair. The new versions of these definitions must be fully syntactic; we must consider only the structure of a procedure and disregard its execution.

All of Kosterec's examples of inconsistency are rooted in (Double) Execution. A Trivialization-bound variable can become free due to the application of Double Execution; new variables that are apparently free for substitution can emerge by executing Double Execution. This is something we do not want to happen. It must be determinate whether a variable occurs free or bound. The form of the solution is that one can substitute only for those variables that are 'syntactically' free, which excludes Trivialization-bound variables. This is the way Tichý chose, and we are going to adhere to Tichý's original definition – with one important extension. There is a ^{20}-elimination rule, i.e., $^{20}C \approx C$, which is valid for any procedure C, and this rule should be accounted for. But ^{20}-elimination concerns only those cases where the rule can be applied correctly: it must not 'propagate' DoubIe Execution inside a given procedure, thus making variables free during execution. This needs to be blocked. The goal of our research is to define correct collision-less substitution based on an updated definition of a variable free for substitution that would account for the correct application of the ^{20}-elimination rule. Another, no less important, goal would be to prove the Compensation Principle, hence the consistency of this version of TIL.

We are solving a problem germane to TIL, but the problem and its solution are relevant to any kind of procedural semantics that comes with a pair of dual procedures (operations) where one cancels out the effect of the other, and which allows more than one kind of variable-binding (in TIL, both λ-binding and Trivialization-binding), such that it may be indeterminate whether an occurrence of a variable is free or bound.

[2]In TIL, we usually do not talk about the syntax and semantics of the TIL language of procedures. It is because this language comes with an interpreted syntax (our ideography). The terms are isomorphic with the procedures they denote. Hence, we speak directly about the structure of those procedures, which corresponds to the 'syntactic' level. However, in this paper, we will stick to the terms 'syntactic' and 'semantic' to make the exposition easier to read for those who are acquainted with traditional model-theoretic λ-calculi rather than TIL.

[3]See, for instance, Duží and Kosterec (2017) or Duží (2019).

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γ-admissibility in first-order relevant logics

NICHOLAS FERENZ
Institute of Computer Science, Czech Academy of Sciences, Czechia
e-mail: ferenz@cs.cas.cz

THOMAS M. FERGUSON
Institute of Computer Science, Czech Academy of Sciences, Czechia
e-mail: tferguson@gradcenter.cuny.edu

The rule of disjunctive syllogism and its cousin the rule-of-proof γ have played a significant role in the history of relevant logic. Disjunctive syllogism is rejected in relevant logics. This rejection is necessary, given their philosophical motivations. In particular, disjunctive syllogism is a common route (for which we commonly thank C.I. Lewis) to explosion: that a contradiction implies everything. The rule of γ, however, is disjunctive syllogism presented as a rule of proof: if the premises of the rule are *theorems* (in a pure presentation with no proper axioms), then so is the conclusion. The rule of necessitation in modal logic is similar. Necessitation is invalid when applied to an arbitrary premise, but valid (in many modal logics) when applied to theorems. We denote the rule γ as follows (where $\Rrightarrow$ indicates a rule of proof):

(γ) $A, \neg A \vee B \Rrightarrow B$

In relevant logic, the admissibility of γ has been a significant topic. While γ is not included in the axiomizations of relevant logics for both practical and philosophical reasons (see Urquhart (2016) for historical details), and while γ is not a derivable rule, it remains an important question in relevant logics to show whether or not it is a safe (admissible) rule to use. That is, whether adding γ would introduce any new theorems.

The admissibility of γ has been thoroughly explored in the setting of modal relevant logics (with relational semantics), as in several papers including Seki (2011), as well as the propositional setting. However, the first-order setting has yet to receive much attention. There have been demonstrations of γ's admissibility in some first-order relevant logics. In Meyer Dunn & Leblanc (1974), the method of normal algebras is used to show γ is admissible in **RQ**. We aim to provide a much more general account of γ admissibility in first-order relevant logics, using the method of normal (frame-based) models.

In the first-order setting, the axiom of extensional confinement is often employed. In particular, e.g., the logic **RQ** extends **QR** by adding this axiom. In a form that uses the universal quantifier, using the notation C^x to denote that x does not occur free in C^x, the axiom of extensional confinement is as follows:

(EC) $\forall x(C^x \vee B) \to (C^x \vee \forall xB)$

The inclusion of this axiom has interesting consequences, as it makes derivable a certain rule-form that is often required for γ-admissibility: the disjunctive form of a rule. In particular, we obtain the following disjunctive rule as derivable:

(d-$\forall$I) $C^x \vee B \Rrightarrow C^x \vee \forall xB$

(That is, in logics with a rule of universal generalization and modus ponens.) This is a disjunctive variant of the rule of universal generalization. Note well, however, the restriction on the subformula C^x is required, otherwise we could derive dangerously invalid formulas such as $\forall xCx \vee \forall x\neg Cx$.

In this work, we

1. determine which first-order relevant logics admit the rule γ
2. discuss the possibility of extending out results to modal relevant logics
3. discuss the philosophical implications of these results

For (1), we employ the technique of normal models. The models are based on a set W of points (situations or, by analogy, possible world), a set $N \subseteq W$ of logically normal points, a ternary relation R modeling the conditional, and a $*$ function modeling negation. Notable, each point has a negation- or star-pair such that $\alpha \vDash \neg A$ iff $\alpha^* \nvDash A$. The technique of normal models is used for propositional relevant logics, e.g., in Routley, Meyer, Plumwood, & Brady (1982), and for modal relevant logics in Seki (2011). (Note that in Meyer Dunn & Leblanc (1974) the method of *normal algebras* is similar to normal models, and we may use this similarity to our advantage.) A model is *normal* when there is situation α (in N) such that $\alpha^* = \alpha$. That is, there is a point where negation behaves 'classically', as in $\alpha \vDash \neg A$ iff $\alpha \nvDash A$.

The technique of normal models shows that every model can be normalized: that is, transformed into a normal model (by adding a point in the frame) such that the logic remains sound and complete w.r.t. the class of normal models. If every model can be normalized in such a way, then γ is shown to be admissible. This is because (i) the rule's premises being valid in the model imply they are true at the normal point α, (ii) no sentence can be both true and false at α, and so (iii) if A is true, $\neg A$ is not true, and thus (iv) $\neg A \vee B$ is true at α only if B is.

We apply the technique of normalizing models to the Mares-Goldblatt style general frames for relevant logics (introduced in Mares & Goldblatt (2006) for first-order extensions of **R**, and generalized in Ferenz (2023) to a wide range of first-order (modal) relevant logics). The Mares-Goldblatt style semantics relies on using general frames, wherein not every hereditary set of points/situations can serve as the truth set of a formula. That is, not every hereditary set of points is *admissible*. In the Mares-Goldblatt semantics, the truth-set (a.k.a (UCLA) proposition) of a universally quantified formula $\forall x A x$ is the largest *admissible* set that is contained in the truth set of each instance. The main result we prove is the identification of a set of first-order relevant logics, defined axiomatically, in which γ is admissible. Particular attention is given to the first-order axioms and rules and their disjunctive counterparts.

For aim (2), the philosophical implications, a particular focus is given to (EC), with its relation to disjunctive rules. In the setting of the Mares-Goldblatt semantics, the semantic condition for (EC) is the only reason why the semantics employed admissible propositional functions in addition to admissible proposition. We hope to provide additional philosophical insight into the axiom of extensional confinement and its semantic condition given its role in γ admissibility.

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Two systems of representational analytic containment

THOMAS M. FERGUSON
Institutes of Philosophy and Computer Science, Czech Academy of Sciences, Czechia
e-mail: tferguson@gradcenter.cuny.edu

FILIP JANKOVEC
Institute of Computer Science, Czech Academy of Sciences and Charles University, Czechia
e-mail: jankovec@cs.cas.cz

Richard Angell's *logic of analytic containment* (AC) of (Angell, 1977) is a subclassical logic introduced as a model of the notion of *synonymy* between propositions. AC is properly included in the relevant logic FDE and makes up an important part of the sub-FDE landscape. Axiomatically, AC may be defined as follows:

A_0	$A \wedge B \vdash A$	A_5	$A \vee (B \vee C) \dashv\vdash (A \vee B) \vee C$
A_1	$A \dashv\vdash A$	A_6	$(A \wedge B) \dashv\vdash A \vee B$
A_2	$A \dashv\vdash A \wedge A$	A_7	$(A \vee B) \dashv\vdash A \wedge B$
A_2'	$A \wedge B \dashv\vdash B \wedge A$	A_8	$(A \wedge B) \vee (A \wedge C) \vdash A \wedge (B \vee C)$
A_3	$A \wedge (B \wedge C) \dashv\vdash (A \wedge B) \wedge C$	A_9	$(A \vee B) \wedge (A \vee C) \vdash A \vee (B \wedge C)$
A_4	$A \dashv\vdash A \vee A$	A_{10}	$A \wedge (B \vee C) \vdash (A \wedge B) \vee (A \wedge C)$
A_4'	$A \vee B \dashv\vdash B \vee A$	A_{11}	$A \vee (B \wedge C) \vdash (A \vee B) \wedge (A \vee C)$

$$
\begin{array}{ll}
R_1 & A \vdash B, B \vdash C \Rightarrow A \vdash C \\
R_2 & A \vdash B, B \vdash A \Rightarrow A \wedge C \vdash B \wedge C \\
R_3 & A \vdash B, B \vdash A \Rightarrow A \vee C \vdash B \vee C
\end{array}
$$

$A \vdash_{\mathsf{AC}} B$ is the consequence relation determined by A_0–A_{11} and rules R_1–R_3 Alongside many-valued semantics due to Ferguson in (Ferguson, 2016), Kit Fine's (Fine, 2016) provides a model in which AC corresponds to the *containment* of the *content* of one proposition in another. The models are state spaces understood as spaces of verifiers/falsifiers:

A *state space model* is a tuple $\langle S, \sqsubseteq, |\cdot|^+, |\cdot|^- \rangle$ where $\langle S, \sqsubseteq \rangle$ is a state space (*i.e.* $\sqsubseteq$ is a partial ordering on S and every subset of S has a least upper bound with respect to $\sqsubseteq$) and valuations $|\cdot|^+$ and $|\cdot|^-$ are functions mapping atoms to *nonempty* subsets of S such that $|p|^+$ and $|p|^-$ are complete. Truth conditions are given in terms of verification and falsification:

$$
\begin{array}{l}
s \Vdash^+ p \text{ if } s \in |p|^+ \\
s \Vdash^+ \neg A \text{ if } s \Vdash^- A \\
s \Vdash^+ A \wedge B \text{ if } \exists t \sqcup u = s \text{ s.t. } t \Vdash^+ A \;\&\; u \Vdash^+ B \\
s \Vdash^+ A \vee B \text{ if } s \Vdash^+ A,\ s \Vdash^+ B, \text{ or } s \Vdash^+ A \wedge B \\
\\
s \Vdash^- p \text{ if } s \in |p|^- \\
s \Vdash^- \neg A \text{ if } s \Vdash^+ A \\
s \Vdash^- A \wedge B \text{ if } s \Vdash^- A,\ s \Vdash^- B, \text{ or } s \Vdash^- A \vee B \\
s \Vdash^- A \vee B \text{ if } \exists t \sqcup u = s \text{ s.t. } t \Vdash^- A \;\&\; u \Vdash^- B
\end{array}
$$

Semantic consequence for AC is given in terms of the Egli-Milner ordering: Let $T, U \subseteq S$.

- $T \sqsupseteq U$—'T *subsumes* U'—if for all $t \in T$, there is a $u \in U$ such that $u \sqsubseteq t$
- $U \sqsubseteq T$—'U *subserves* T'—if for all $u \in U$ there is a $t \in T$ such that $u \sqsubseteq t$

For $T, U \subseteq S$, T *contains* U $(T > U)$ if $\begin{cases} T \sqsupseteq U, \text{ and} \\ U \sqsubseteq T \end{cases}$

Then the content of a formula can be defined as a closure operation on its set of verifiers:

The set $[A]^+$—the *replete content* of A—is $\lceil A \rceil^+_*$, *i.e.*, the convex closure of the set $\{s \mid s \Vdash^+ A\}$.

In (Fine, 2016), Fine provides two equivalent *unilateral* and *bilateral* definitions of consequence in his state space semantics, both of which correspond to provability in AC.

[Fine] $A \vdash_{\mathsf{AC}} B$ iff in any strong state space model, $[A]^+ > [B]^+$

[Fine] $A \vdash_{\mathsf{AC}} B$ iff in any strong state space model, $\begin{cases} [A]^+ > [B]^+ & \text{, and} \\ [B]^- \subseteq [A]^- \end{cases}$

A weaker notion of analytic equivalence was introduced by Fabrice Correia as the equivalential logic FE in (Correia, 2016). Correia's system resists a formulation as a consequence relation, however. There are independent reasons for being interested in an understanding of *containment* in FE, *e.g.*, allowing us to study its role in the sub-FDE landscape, motivating correlated systems of *factual containment*. We define two systems that relate to FE appropriately:

$A \vdash_{\mathsf{RAC}_1} B$ is the consequence relation determined by A_0–A_{10} and rules R_1–R_3

$A \vdash_{\mathsf{RAC}_2} B$ is the consequence relation determined by A_0–A_9 and rules R_1–R_3

Interestingly, these systems can be understood as a *perspectival* variation on AC in which the content of a proposition is restricted to the perspective of an arbitrary state s:

For a state s and formula A, let $\lceil A \rceil\restriction_s$—the *$s$-perspectival content of* A—be defined as $\{t \in S \mid t \sqsubseteq \lceil A \rceil \text{ and } t \sqsubseteq s\}$.

Interestingly, when validity is judged over such perspectival contents, Fine's unilateral and bilateral semantics come apart:

$A \vdash_{\mathsf{RAC}_1} B$ iff in any strong state space model and state s, $[A]^+\restriction_s > [B]^+\restriction_s$

$A \vdash_{\mathsf{RAC}_2} B$ iff in any strong state space model, $\begin{cases} [A]^+\restriction_s > [B]^+\restriction_s & \text{, and} \\ [B]^-\restriction_s \subseteq [A]^-\restriction_s \end{cases}$

And to return to Correia's factual equivalence, we have a consequence relation that is in the spirit of the equivalential FE:

$\vdash_{\mathsf{FE}} A \leftrightarrow B$ iff $A \vdash_{\mathsf{RAC}_1} B$ and $B \vdash_{\mathsf{RAC}_1} A$

Building off of observations made in (Ferguson, 2017), this provides an interpretation of Correia's FE that is compatible with Definition 2.

Philosophically, the systems RAC_1 and RAC_2 can be justified by appeal to the irreducibly perspectival nature of mental activity. The requirement that every proposition has verifiers and falsifiers is implausible as a property of *human minds*, *i.e.*, reasoners do not have the infinite capacity and experience to maintain among their cognitive resources verifiers/falsifiers for every possible sentence. Thus, as far as a reasoner's *internal* logic of synonymy—the logic of analytic containment of their *representations* of the world—the model-theoretic pictures of RAC_1 or RAC_2 are better fits than AC itself.

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Genealogical containment in Parry-style logics

THOMAS M. FERGUSON
Institutes of Philosophy and Computer Science, Czech Academy of Sciences, Czechia
e-mail: tferguson@gradcenter.cuny.edu
JITKA KADLEČÍKOVÁ
Institute of Computer Science, Czech Academy of Sciences and Palacky University Olomouc, Czechia
e-mail: kadlecikova.jitka@gmail.com

Recent developments of Parry-style logics have been introduced in which new developments in the theory of topic are applied to Parry's PAI (of Parry (1933(@)) in order to generate more refined systems of containment logic. In particular, we focus on the system of *conditional-agnostic analytic implication* CA/PAI introduced in Ferguson (2023). The system was motivated by an acknowledgement both that conditionals can transform topic and the regularities of such transformations can vary wildly depending on the type of conditional whose topic is being determined. CA/PAI thus characterizes a Parry-style system making minimal assumptions about the topic assigned to any given conditional in the language. The system clearly should impose stricter requirements on containment than Parry's favored condition but such an investigation was not undertaken in Ferguson (2023).

Model theoretically, CA/PAI models can be defined by modifying Fine's models of Fine (1986) in the following way:

A CA/PAI Fine model is a tuple $\langle W, R, \mathcal{T}, \oplus, \multimap, v, t, h\rangle$ such that:

- $\langle W, R\rangle$ is an S4 Kripke frame
- For each $w \in W$, $\langle \mathcal{T}_w, \oplus_w\rangle$ is a join semilattice
- v is a valuation from atomic formulae to W
- For each $w \in W$, t_w is a function mapping atomic formulae to $\mathcal{T}_w$
- For each $w \in W$, $\multimap_w$ is a binary function from $\mathcal{T}_w \times \mathcal{T}_w \to \mathcal{T}_w$
- For all w, w' such that wRw', $h_{w,w'} : \mathcal{T}_w \to \mathcal{T}_{w'}$ is a homomorphism such that:
  - for atoms p, $h_{w,w'}(t_w(p)) = t_{w'}(p)$
  - $h_{w,w'}(a \oplus_w b) = h_{w,w'}(a) \oplus_{w'} h_{w,w'}(b)$
  - $h_{w,w'}(a \multimap_w b) = h_{w,w'}(a) \multimap_{w'} h_{w,w'}(b)$

The topic assignment function t_w is extended through the language:

- $t_w(\neg\varphi) = t_w(\varphi)$
- $t_w(\varphi \star \psi) = t_w(\varphi) \oplus_w t_w(\psi)$ for $\star$ extensional
- $t_w(\varphi \to \psi) = t_w(\varphi) \multimap_w t_w(\psi)$

Truth conditions are defined recursively:

- $w \Vdash p$ if $w \in v(p)$
- $w \Vdash \neg\varphi$ if $w \nVdash \varphi$
- $w \Vdash \varphi \wedge \psi$ if $w \Vdash \varphi$ and $w \Vdash \psi$
- $w \Vdash \varphi \to \psi$ if $\begin{cases} \text{for all } w' \text{ such that } wRw', \text{ if } w' \Vdash \varphi \text{ then } w' \Vdash \psi \\ t_w(\psi) \leq_w t_w(\varphi) \end{cases}$

We can define validity in $\mathsf{CA/PAI}$ in the standard way.

Informally, we can approach the $\multimap$ function as encapsulating the *agnosticism* of $\mathsf{CA/PAI}$, allowing the model nearly complete freedom to assign a topic to a conditional. Despite this freedom, the device is nevertheless strong enough to ensure some interesting variations of the Proscriptive Principle hold. To explore further, we will need to avail ourselves of some discussion of notation. First, consider the following definition of a formula's *armature*:

The *armature* of a sentence φ is defined as follows:

- $\mathcal{L}_{\rightarrow}(p) = \{p\}$
- $\mathcal{L}_{\rightarrow}(\neg\varphi) = \mathcal{L}_{\rightarrow}(\varphi)$
- $\mathcal{L}_{\rightarrow}(\varphi \star \psi) = \mathcal{L}_{\rightarrow}(\varphi) \cup \mathcal{L}_{\rightarrow}(\psi)$ for $\star$ extensional
- $\mathcal{L}_{\rightarrow}(\varphi \rightarrow \psi) = \{\langle \mathcal{L}_{\rightarrow}(\varphi), \mathcal{L}_{\rightarrow}(\psi)\rangle\}$

If considerations of topic inclusion are taken seriously—and the transformative nature of negation and intensional conditionals is acknowledged (as I think they should be)—Anderson and Belnap's thesis that "commonality of meaning in propositional logic is carried by commonality of propositional variables" is too hasty. Rather, to ensure commonality of meaning, we need to track the genealogy—including order—of the applications of the conditional. In order to formalize this intuition of genealogy, we define a class of objects recording a formula's genealogy:

An *intensional genealogy* of an occurrence of an formula ψ in a formula φ is a string defined recursively:

- If $\varphi = \psi$ then $(\varphi[\psi]) = \langle\rangle$
- $(\neg\varphi[\psi]) = (\varphi[\psi])$
- $(\varphi[\psi] \star \xi) = (\xi \star \varphi[\psi]) = (\varphi[\psi])$ for $\star \in \{\wedge, \vee\}$
- $(\varphi[\psi] \rightarrow \xi) = (\varphi[\psi])\mathtt{L}$
- $(\xi \rightarrow \varphi[\psi]) = (\varphi[\psi])\mathtt{R}$

Essentially, $(\varphi[\psi])$ provides a sort of *provenance* of ψ, *i.e.*, a record of the history of applications of the intensional conditional to the subformula—including the matter of whether ψ was included as antecedent or consequent. Given the foregoing definitions, we have enough material to provide an appropriate refinement of the Parry's original containment property.

A logic satisfies the *genealogical Proscriptive Principle* if for all theorems $\varphi \rightarrow \psi$, every atomic formula appearing in ψ appears in φ with the same geneaology.

We can now state our fundamental lemma:

$\vdash_{\mathsf{CA/PAI}} \varphi \rightarrow \psi$ only if $\mathcal{L}_{\rightarrow}(\psi) \subseteq \mathcal{L}_{\rightarrow}(\varphi)$

Lemma allows us to establish the main theorem concerning the genealogical Proscriptive Principle:

$\mathsf{CA/PAI}$ observes the genealogical Proscriptive Principle.

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Reconsidering identity

RENE GAZZARI
University of Minho, Portugal
e-mail: elbron@gmx.net

One characteristic feature of Gentzen's Natural Deduction (Gentzen, 1934) is that all logical operators (connectives and quantifiers) have their specific introduction and elimination rules; the introduction rules are considered (at least from the proof-theoretic semantics point of view) as the definition of the meaning of the associated operator and the elimination rules are required to be in harmony with the respective introduction rule.

Usually, the identity relation is considered to be a logical concept. But the usual Natural Deduction inference rules associated with the equality symbol seem not to fit well into the introduction-elimination schema (see figure 1 for the formulation of some standard rules). In particular, when considering the reflexivity rule (I_1) as introduction and the congruence rule for formulae (I_5) as corresponding elimination rule, then the rules are not in harmony (see, for example, Read (Read, 2004)).

Motivated by calculations occurring in real mathematical proofs (affecting mathematical objects as numbers and sets), Gazzari proposed the calculus of Natural Calculation (Gazzari, 2021), an extension of Natural Deduction by proper term rules (see figure 2 for some example rules) permitting, in the presented version, the formal representation of equality calculations.[1] Some of the new term rules can be identified clearly as introduction and elimination rules for the equality symbol. This observation motivates the reconsideration of the identity relation based on the new term rules and from a proof-theoretic semantics point of view; as a side effect, the reconsideration yields some insight into the nature of the usual identity rules.[2]

By proposing harmonic introduction and elimination rules for the equality, our work stays clearly in tradition of Read's approach (Read, 2004); but in contrast to him, we do not have to consider second order logic. As another related work, we have to mention Klev's analysis of the harmony of identity (Klev, 2019). But despite of similarities caused by the common topic, his analysis differs in many details, in particular due to his distinction between identity and definitional identity.

Adapting Prawitz's inversion principle for logical operators (Prawitz, 1965) to the case of term rules, we will argue that the pair of introduction and elimination rule for the equality symbol is, indeed, in harmony, provided we accept some variations of the canonical proofs as unproblematic (which we already have to do in the case of the usual logical operators).

The harmony is slightly disturbed in the presence of relation symbols in the underlying first order language. In this case, a complete calculus requires an additional (term) rule (similar to the congruence for formulae rule) affecting, at least, these relation symbols. Our analysis shows that there is reason to classify this rule outside of the schema of introduction and elimination rules as a substitution rule.

In constructive logic, the meaning of the logical operators is explained by the BHK interpretation of logic. We suggest to extend this interpretation by a clause for the equality symbol:

[1] The term rules are easily modified to permit the representation of other types of calculations, as, for example, smaller-than calculation.

[2] Indrzejczak (Indrzejczak, 2021) carried over our idea of treating terms on a par with formulae to the sequent calculus, where he undertakes similar investigations on term rules for the identity.

Figure 1: Some Standard Identity Rules

$$\frac{}{t = t}\,(I_1) \quad ; \quad \frac{A(t) \qquad s = t}{A(s)}\,(I_5)$$

Figure 2: Some Term Rules

$$\frac{r(t) \qquad t = s}{r(s)}\,(E^{+}_{=}) \quad ; \quad \frac{r(t) \qquad s = t}{r(s)}\,(E^{-}_{=}) \quad ; \quad \frac{\begin{matrix}[t] \\ s\end{matrix}}{t = s}\,(I_{=})$$

- An equation $t = s$ is the statement that there is a an equality calculation from the term t to the term s.

As the meaning of the equality symbol is not (completely) determined by its introduction rule (which seems to be different in the case of the other logical operators), we suggest another clause dealing with the elimination rule:

- A calculation step from a term t to a term s is the justified replacement of some occurrences of a term r_0 in t by a term r_1, resulting in s.

  A justification is an equation $r_0 = r_1$ (or an equality calculation from r_0 to r_1).

As the justifications required for a calculation step are assumptions or derived in an elimination part of the derivation, the second clause is not circular.

Having the constructive reading of equations in mind, we can analyse the usual identity rules as meta-rules (similar to the sequents of the sequent calculus) asserting the existence of specific kinds of calculations. In the case of transitivity, for example, this is the assertion of the existence of a calculation from a term t to a term r provided that there is a calculation from t to a term s as well as from s to r.

Furthermore, we can consider, on the base of the constructive reading of equality, to embed the calculus of Natural Calculation into usual Natural Deduction with standard identity rules. It is worth mentioning that this attempt fails, as standard Natural Deduction is not able to represent the introduction of equality rule of Natural Calculation. In other words: Natural Deduction cannot distinguish between a calculation from t to s and a derivation, in which the calculation is evaluated and in which the result $t = s$ is inferred. For this reason, we do not consider any of the standard identity rules as an introduction rule for the equality.

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Decidability of intuitionistic S4

MARIANNA GIRLANDO[1]
University of Amsterdam, Netherlands
e-mail: m.girlando@uva.nl

ROMAN KUZNETS[2]
TU Wien, Austria
e-mail: roman@logic.at

SONIA MARIN
University of Birmingham, United Kingdom
e-mail: s.marin@bham.ac.uk

MARIANELA MORALES
Inria Saclay & École Polytechnique, France
e-mail: marianelamoralese@gmail.com

LUTZ STRASSBURGER
Inria Saclay, France
webpage: https://www.lix.polytechnique.fr/~lutz/

Logic S4 was among the first modal logics in the "modern" logical tradition, the fourth system of C. I. Lewis. Its common axiomatic formulation is due to Gödel. The commonly used semantics for it as the logic of all reflexive and transitive Kripke frames is also at least half a century old. Little remains unknown about it, and it enjoys most properties desirable of a well-behaved logic. In particular, its decidability was shown by Ladner (1977).

The propositional basis of S4 is classical, so it is natural to study what happens when it is replaced by intuitionistic propositional logic (IPL). While the transition is not entirely deterministic, we focus here on what eventually became known as *intuitionistic modal logics* in the tradition of Fischer Servi (1984) and Plotkin and Stirling (1986), which were investigated in detail by Simpson (1994). While it is reasonable to expect that intuitionistic reasoning makes things more complex compared to classical one, this is a priori more likely to cause the increase in complexity than to lead to an undecidable logic. Thus, it is all the more surprising that the problem of decidability of IS4, i.e., of intuitionistic S4, remained open since it was formulated by Simpson (1994). We finally solve this question positively: IS4 is decidable.

The langugage of ***logic*** IS4 is $A ::= \bot \mid a \mid (A \wedge A) \mid (A \vee A) \mid (A \supset A) \mid \Box A \mid \Diamond A$ where $a \in \mathcal{A}$ is an atomic formula (note that, unlike for S4, modalities $\Box$ and $\Diamond$ are independent). Its axiom system is obtained by extending any standard axiom system for IPL with

k_1: $\Box(A \supset B) \supset (\Box A \supset \Box B)$ $\quad$ k_2: $\Box(A \supset B) \supset (\Diamond A \supset \Diamond B)$

k_3: $\Diamond(A \vee B) \supset (\Diamond A \vee \Diamond B)$ $\quad$ k_4: $(\Diamond A \supset \Box B) \supset \Box(A \supset B)$ $\quad$ k_5: $\Diamond\bot \supset \bot$

$\mathsf{4}$: $(\Diamond\Diamond A \supset \Diamond A) \wedge (\Box A \supset \Box\Box A)$ $\quad$ t: $(A \supset \Diamond A) \wedge (\Box A \supset A)$

and the standard necessitation rule. As classical S4, Kripke frames of IS4 are reflexive and transitive, but in the so-called *birelational semantics*:

A ***birelational model*** $\mathcal{M}$ for IS4 is a quadruple $\langle W, R, \leq, V\rangle$ of a set $W \neq \varnothing$ of ***worlds*** equipped with two preorders (i.e., reflexive and transitive relations) — an ***accessibility relation*** R and future relation $\leq$ — and a ***valuation function*** $V \colon W \to 2^{\mathcal{A}}$ satisfying:

(F_1) For all $x, y, z \in W$, if xRy and $y \leq z$, there exists $u \in W$ such that $x \leq u$ and uRz.
(F_2) For all $x, y, z \in W$, if $x \leq z$ and xRy, there exists $u \in W$ such that zRu and $y \leq u$.
(M) If $w \leq w'$, then $V(w) \subseteq V(w')$.

Forcing $\Vdash$ for atomic formulas is determined by the valuation function: $\mathcal{M}, w \Vdash a$ iff $a \in V(w)$, with $\mathcal{M}, w \nVdash \bot$. It is recursively extended to all formulas as follows:

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$\mathcal{M}, w \Vdash A \wedge B$	iff	$\mathcal{M}, w \Vdash A$ and $\mathcal{M}, w \Vdash B$;
$\mathcal{M}, w \Vdash A \vee B$	iff	$\mathcal{M}, w \Vdash A$ or $\mathcal{M}, w \Vdash B$;
$\mathcal{M}, w \Vdash A \supset B$	iff	for all w' with $w \leq w'$, if $\mathcal{M}, w' \Vdash A$, then $\mathcal{M}, w' \Vdash B$;
$\mathcal{M}, w \Vdash \Box A$	iff	for all w' and u with $w \leq w'$ and $w'Ru$, we have $\mathcal{M}, u \Vdash A$;
$\mathcal{M}, w \Vdash \Diamond A$	iff	there exists u such that wRu and $\mathcal{M}, u \Vdash A$.

Theorem (Fischer Servi (1984); Plotkin and Stirling (1986))**.** A formula A is a theorem of IS4 if and only if A is valid in every birelational model for IS4.

Our proof of decidability of IS4 is proof-theoretical. A proof search is performed in a suitable analytic sequent-like calculus for IS4. If the proof search is successful in finding a proof, the formula in question is derivable. Otherwise, a failed proof search provides sufficient information to construct a countermodel. The difficulties in applying this method to IS4 are not new either. It is typical that a naive proof search for a logic with transitive Kripke frames does not terminate. Thus, loop-checks are used for both S4 (w.r.t. transitive R) and IPL (w.r.t. transitive $\leq$) to stop the naive proof search. A non-terminating naive proof search is bound to enter into a loop due to the subformula property, which ensures a global bound on the number of sequents that can appear in a proof search. When that happens, a countermodel can be constructed by emulating the algorithm loop by an appropriate R-loop for S4 or $\leq$-loop for IPL.

The unique challenges of IS4 are due to the fact that the two sources of repetition can interact, creating a possibility of a proof search neither terminating nor repeating any sequents. To overcome this problem we use a fully labelled sequent calculus (see Maffeziolli et al. (2013); Marin et al. (2021)) with relational atoms for both relations R and $\leq$, where R-loops can be represented on a sequent level. Since labelled sequent rules do not ordinarily create such loops, we incorporate several loop-checks into the proof search algorithm by adding new rules for creating R-loops. This R-loop-enabled proof search still does not guarantee sequent repetition, forcing us to formulate a more complex loop-check condition with respect to $\leq$-loops: the proof search is stopped if the latest sequent can be *emulated* by an earlier sequent. The soundness of the new R-loop-creating rules is proved by a non-trivial *unfolding* algorithm that converts derivations with R-loops into proper loop-free derivations by creating multiple duplicates of each loop. Thus, this loop-augmented proof search provides a decision procedure for IS4.

Theorem. Logic IS4 is decidable.

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No cause for collapse

DUSTIN GOOSSENS
Ruhr-Universität Bochum/Institut für Philsoophie I, Germany
e-mail: dustin.goossens@rub.de

ANDREW TEDDER
Ruhr-Universität Bochum/Institut für Philsoophie I, Germany
e-mail: ajtedder.at@gmail.com

There are two different types of collapse arguments against logical pluralism. The first one will be called 'upward collapse' because it results in the collection of logics endorsed by the pluralist to collapse into the strongest one.[1] The second one will, in the same spirit, be called 'downward collapse' and is supposed to result in an endorsement of only the weakest admissible logic.[2] Both arguments, however, (oftentimes tacitly) depend on the order-theoretic properties of the collection of admissible logics to have a strictly strongest and, respectively, a strictly weakest logic. So, taking the collection of admissible logics to form a lattice, it seems to be assumed in various collapse arguments that the collection of admissible logics will include logics which are *lattice joins* and *lattice meets*, to which the pluralist is then argued to be *really* committed. In the following, this order-theoretic assumption will be dropped and the prospect for avoiding collapse arguments in this way will be analyzed from a logical as well as philosophical point of view.

Defining logics by a Tarksian consequence relation over a fixed language, this family of logics is known to form a complete lattice. This allows us to define a logic as stronger when it encompasses the compared logic as a subset:

$$\mathbf{L}_1 \leq \mathbf{L}_2 \iff \vdash_{\mathbf{L}_1} \subseteq \vdash_{\mathbf{L}_2}$$

This order, as has been discussed in (Wójcicki, 1988), is a complete lattice in that each set of logics $\{\mathbf{L}_i\}_{i\in I}$ has a *join* $\bigvee\limits_{i\in I}\mathbf{L}_i$ and a *meet* $\bigwedge\limits_{i\in I}\mathbf{L}_i$. The meet is defined straightforwardly as set intersection:

$$\bigwedge_{i\in I}\mathbf{L}_i = \bigcap_{i\in I}\mathbf{L}_i$$

but the join is not set union (for reasons of transitivity), but rather requires the more involved definition:

$$\bigvee_{i\in I}\mathbf{L}_i = \bigwedge\{\mathbf{L} \mid \bigcup_{i\in I}\mathbf{L}_i \subseteq \mathbf{L}\}$$

The main point of this paper is to propose a formal way of avoiding collapse arguments by motivating meet- or join-incomplete collections of logics.

The upward collapse argument was developed by several authors and is likely to be the most prominent argument against logical pluralism. Its general idea is that if two logics disagree about the validity of an argument, the normativity of logic obliges us to reason using the stronger logic in any case of disagreement, thus leaving the weaker logic normatively ineffective. This is then taken to rule out the weaker logic's claim to be an admissible logic and, in turn, collapses upward into monism about the strongest logic. Since each collection of

[1] Examples of the upward collapse argument can be found in (Priest, 2006; Read, 2006; Keefe, 2014)

[2] An example of the downward collapse is provided in (Steinberger, 2019)

logics has a join, the proposed solution to the upward collapse must motivate endorsing two logics but not their join in a principled way. It is proposed that this motivation can be provided by the join losing certain important properties that each of the weaker logics still exhibits. An easy example of this is found in the literature is endorsing classical logic and a contra-classical logic (e.g. Abelian logic).[3] The join of these logics is the trivial logic, which does not exhibit important properties that the weaker logics still do. A more tangible example is endorsing the relevance logics **R** and **TM**, which both fulfill the variable-sharing criterion, while their join, **RM**, does not. A staunch relevance logician can thus be a pluralist about **R** and **TM** by deeming variable-sharing to be a necessary condition for true logicality.[4]

Avoiding licensing the meet of a collection of admitted logics to itself be admissible is trickier to motivate as, by definition, the meet of a collection of logics *will not* invalidate any arguments validated by all the 'meeted' logics. However, it is still possible for the meet of two logics to lose systematic properties exhibited by each of the 'meeted' logics. An example of this are the logics **J** and **RW** that each exhibit the *disjunction property* while it can be shown that their meet $\mathbf{J} \wedge \mathbf{RW}$ fails to do so. Another reason for a pluralist to not admit the meets of their logic among their collection of admissible logics can be seen in various versions of *goal-oriented* pluralism as for instance developed by Shapiro (2014); Blake-Turner and Russell (2018); Commandeur (2022); Cook (2023). The general idea in these cases is that each of the endorsed logics serves a certain goal, while the logic that is their meet is likely to not help to attain any of these specifically logical goals anymore.

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[3]For example discussed in (Read, 2006)

[4]Other examples of this besides non-triviality and relevance are different ways of preserving paraconsistency whose join again licenses ECQ, or logics (like the intuitionistic logic **J** and the *contraction-free* relevant logic **RW**) that both exhibit the disjunctive property while their join fails to do so.

Logic of quantifier shifts

ROSALIE IEMHOFF
Utrecht University/Department of Philosophy, The Netherlands
e-mail: r.Iemhoff@uu.nl
RAHELEH JALALI
Utrecht University/Department of Philosophy, The Netherlands
The Czech Academy of Sciences/Institute of Computer Science, Czechia
e-mail: rahele.jalali@gmail.com

Skolemization is a method to remove strong quantifiers (i.e., positive occurrences of the universal quantifier and negative occurrences of the existential quantifier) from a first-order formula, and replace them with fresh function symbols. It is a well-known fact that Skolemization is sound and complete with respect to the classical predicate logic, CQC, while it is not the case for the intuitionistic predicate logic, IQC. Several studies have been done on the Skolemization in intermediate logics, including introducing alternative methods (Baaz, Iemhoff, 2016; Iemhoff, 2010). To explain more, let us first present the following formulas, that we call the *quantifier shifts*:

1. (Constant Domain) $\forall x(A(x) \vee B) \to \forall x A(x) \vee B$ (CD)
2. (Quantifier Switch) $(\forall x A(x) \to B) \to \exists x(A(x) \to B)$ (SW)
3. (Existential Distribution) $(B \to \exists x A(x)) \to \exists x(B \to A(x))$ (ED)

where $A(x)$ and B are formulas in the first-order language and the variable x is not free in B. None of these formulas are provable in IQC. However, in CQC, both these formulas and their converses are provable. One may suspect that the failure of the quantifier shifts is the reason why Skolemization fails in IQC. Therefore, it is natural to ask what happens if we add the quantifier shifts to IQC. Does the resulting logic have Skolemization? If not, for which class of formulas does the Skolemization hold? These questions build the motivation of the present research study that focuses first on investigating the *logic of quantifier shifts* and then its Skolemization. This talk is devoted to the first part of the study.

Denote the logic $\mathsf{IQC} + \{\mathrm{CD}, \mathrm{SW}, \mathrm{ED}\}$ by QFS. In the following, we will investigate the properties of this logic and its *fragments*, state the main results of this ongoing research, and sketch some of the proofs.

Definition 1 (Kripke frames and models)**.** (Mints, 2000, Chapter 14) A *Kripke frame* for IQC is a triple (W, R, D), where $W \neq \emptyset$ is a set of worlds, R is a binary reflexive and transitive relation over W, and D is a function assigning to each $w \in W$ a non-empty set $D(w)$, called the *domain* of w, such that if wRw' then $D(w) \subseteq D(w')$. A *Kripke model* for IQC is a quadruple (W, R, D, V) where (W, R, D) is a Kripke frame and V is a valuation function in its usual sense. A formula A is defined to be *valid* in a frame F, denoted by $F \vDash A$, and valid in a model M, denoted by $M \vDash A$, as usual. A Kripke frame is called *linear* when for any $w, w' \in W$ either wRw' or $w'Rw$. We call a Kripke frame *constant domain* when for any $w, w' \in W$, we have $D(w) = D(w')$.

First, let us observe the following easy facts that separate the fragments of QFS that we are interested in:

$$\mathsf{IQC} + \{\mathrm{CD}, \mathrm{SW}\} \nvdash \mathrm{ED} \quad \text{and} \quad \mathsf{IQC} + \{\mathrm{CD}, \mathrm{ED}\} \nvdash \mathrm{SW}.$$

Having this observation, we know that the logics QFS, $\mathsf{IQC}+\{\mathrm{CD},\mathrm{SW}\}$, and $\mathsf{IQC}+\{\mathrm{CD},\mathrm{ED}\}$ are all distinct. The following definition introduces rich classes of frames for these three logics.

Definition 2. Consider the following class $\mathscr{F}$ of Kripke frames:

1. Linear, constant domain, finite number of worlds (with finite/infinite domains)

2. Linear, constant domain, infinite number of worlds with finite domains.

3. Constant domain, and $D(w)$ has exactly one element, for any $w \in W$.

Then, define $\mathscr{F}_{\mathrm{SW}}$ (resp. $\mathscr{F}_{\mathrm{ED}}$) by adding all "linear, constant domain and conversely well-founded (well-founded)" frames to $\mathscr{F}$.

Theorem 3. *For any Kripke frame F:*

1. $F \vDash \mathsf{QFS}$ *if and only if* $F \in \mathscr{F}$.

2. $F \vDash \mathsf{IQC}+\{\mathrm{CD},\mathrm{SW}\}$ *if and only if* $F \in \mathscr{F}_{\mathrm{SW}}$.

3. $F \vDash \mathsf{IQC}+\{\mathrm{CD},\mathrm{ED}\}$ *if and only if* $F \in \mathscr{F}_{\mathrm{ED}}$.

One may wonder, why in the frame characterization, we always include the axiom CD. The reason simply is that if $F \vDash \mathsf{IQC}+\{\mathrm{SW}\}$ or $F \vDash \mathsf{IQC}+\{\mathrm{ED}\}$, then F must be constant domain and hence adding the axiom CD does not change the frame validity. However, note that it does not mean that the axiom scheme CD is provable in the mentioned logics.

Definition 4. The logic L is called *complete* with respect to the class $\mathscr{C}$ of Kripke frames when

$$L \vdash \varphi \quad \text{if and only if} \quad \mathscr{C} \vDash \varphi,$$

for any formula φ. The logic L is called *frame-complete* if there exists a class $\mathscr{C}$ of Kripke frames such that L is complete with respect to $\mathscr{C}$.

Theorem 5. *The logics* QFS, $\mathsf{IQC}+\{\mathrm{CD},\mathrm{SW}\}$, *and* $\mathsf{IQC}+\{\mathrm{CD},\mathrm{ED}\}$ *are all frame-incomplete.*

Proof. Let us sketch the proof for the case of QFS. To show that QFS is frame-incomplete, we have to prove that for any class $\mathscr{C}$ of Kripke frames for QFS, there exists a formula φ such that $\mathscr{C} \vDash \varphi$ but $\mathsf{QFS} \nvdash \varphi$. We claim that taking an instance of $\varphi = \mathrm{Lin} \vee \mathrm{OEP}$ works, where

$$\mathrm{Lin} := (C \to D) \vee (D \to C) \quad \text{and} \quad \mathrm{OEP} := \exists x A(x) \to \forall x A(x)$$

are the Linearity and One Element Principle schemes. To see why, we show that:

1. If a frame F validates QFS, then it is constant domain. Moreover, F is either linear or its domain is just a singleton. This means that F validates all instances of the axiom scheme $\mathrm{Lin} \vee \mathrm{OEP}$.

2. It is easy to see that there is an instance of the axiom scheme $\mathrm{Lin} \vee \mathrm{OEP}$ such that $\mathsf{QFS} \nvdash \mathrm{Lin} \vee \mathrm{OEP}$.

These two points together prove that QFS is frame-incomplete. ☐

Finally, as the last word in this extended abstract, let us recall the *propositional logic* of a first-order theory T, denoted by $\mathrm{PL}(L)$, as the set of all propositional formulas φ such that for any first-order substitution σ we have $T \vdash \sigma(\varphi)$.

Theorem 6. $\mathrm{PL}(\mathsf{QFS}) = \mathsf{IPC}$.

This theorem intuitively states that, as expected, the quantifier shift formulas have no propositional content and hence adding them to IQC do not change the intuitionistic propositional logical base.

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Stalnaker's thesis and the probability of left-nested conditionals

MARCEL KERTELS
Bielefeld University, Germany
e-mail: mkertels@uni-bielefeld.de

According to Stalnaker's Thesis, the (subjective) probability of an indicative conditional is the conditional probability of its consequent, given its antecedent:

(ST) $P(\phi \to \psi) = P(\psi|\phi)$ for $P(\phi) > 0$.

Despite its intuitive appeal, **(ST)** has been regarded as untenable by many, as it has been shown numerous times to have unacceptable consequences by the so-called triviality results.

Stalnaker (1976) provides an argument against **(ST)** featuring left-nested conditionals. Left-nested conditionals are of the form $(\phi \to \chi) \to \psi$ featuring a conditional as antecedent. As such, they are relatively uncommon in use and seldom elicit clear intuitions. Recently, however, Khoo (2022) has argued that Stalnaker's result is intuitively supported by counterexamples.

Here is *Edgington's coin*, one of the supposed cases against **(ST)**:

> **Edgington's Coin.** We have a coin, x, which is either double-headed or double-tailed. We are fifty-fifty as to each possibility. Similarly, we think it is 0.5 likely that x was flipped, and thus think it is 0.5 likely that x landed heads given that it was flipped. (Khoo, 2022, 154; cf. Jeffrey and Edgington, 1991)

In this scenario, Khoo argues that the following left-nested conditional intuitively has probability 1:

(**1**) If the coin landed heads if it was flipped, it was double-headed. (Khoo, 2022, 154)

However, **(ST)** supposedly allows to derive the following formula for the probability of left-nested conditionals:

PLNC 1. $P((\phi \to \chi) \to \psi) = P(\psi|\phi \wedge \chi) \cdot P(\phi) + P(\neg\phi \wedge \psi)$ for $P(\phi \wedge \chi) > 0$.

PLNC 1 yields as probability of (**1**): $1 \cdot \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$. Hence (**1**) seems to provide a counterexample to **(ST)**.

In this paper, I will argue that neither endorsement nor – as suggested by the case of Edgington's coin – rejection of PLNC 1 should reflect on **(ST)** itself. I will proceed in two steps.

First, a detailed view on the derivation of PLNC 1 will show that PLNC 1 does not follow from **(ST)** and the usual background assumptions about probability – the Kolmogorov axioms and the ratio-formula – alone. Rather, PLNC 1 only follows with an additional and – so I think – highly controversial *independence assumption*:

(IA) $\neg\phi \wedge \psi$ and $\phi \to \chi$ are probabilistically independent. (cf. McGee, 1989, 492ff.; Jeffrey and Edgington, 1991, 180; Jeffrey and Stalnaker, 1994, 37f.)

(IA), I venture, is not tenable, for there is another principle that conflicts with **(IA)**:

(EA) $P(\phi \to \chi|\psi) = 1$ if $\psi \models \phi \to \chi$ for $P(\psi) > 0$.

(EA) follows from the ratio-formula, a background assumption in the derivation of PLNC 1. **(EA)** conflicts with **(IA)** if there are ψ which entail $\phi \to \chi$. One candidate for ψ would be $\phi \to \chi$. Another, non-conditional candidate for ψ might be provided by Edgington's coin.

Second, we can derive an alternative formula for the probability of left-nested conditionals, PLNC 2, whose derivation does not require the independence assumption **(IA)**. PLNC 2 outperfoms PLNC 1 in that it does yield the intuitively correct values in the case of (**1**).

I will argue that if **(ST)** were accepted in full generality, it would yield a different and in some cases more adequate formula for the probability of left-nested conditionals:

PLNC 2. $P((\phi \to \chi) \to \psi) = P(\psi|\phi \wedge \chi) \cdot P(\phi) + P(\neg\phi \wedge \psi) \cdot \left(\frac{P(\psi|\phi\wedge\chi}{P(\psi|\phi)}\right)$ for $P(\phi \wedge \chi) > 0$, $P(\phi \wedge \psi) > 0$.

The derivation of PLNC 2 does not rest on **(IA)**, but on the following – by no means innocuous (see Gibbard, 1981; Fitelson, 2015) – lemma which is a corollary of **(ST)** in its full generality:

lemma 2. $P(\phi \to \psi|\chi) = P(\psi|\phi \wedge \chi)$ for $P(\phi \wedge \chi) > 0$.

Contrary to PLNC 1, PLNC 2 does predict that the probability of (**1**) is 1. Except for the last coefficient $c := \frac{P(\psi|\phi\wedge\chi)}{P(\psi|\phi)}$, bracketed for highlighting, PLNC 1 and PLNC 2 are the same. c we may understand to measure the relevance of χ for ψ in the presence of ϕ. If there is a strong correlation between ψ and χ that is not influenced by ϕ, c is larger than 1 and thereby functions as corrective of the too low value that Khoo identifies as a problem with PLNC 1. In the case of (**1**), this coefficient is 2, such that PLNC 2 yields probability 1 for (**1**) – the intuitively correct value. The same applies the other way round: If χ influences the probability of ψ given ϕ negatively, $c < 1$ and c thereby corrects the value accordingly.

A short conclusion discusses the upshot of these findings. They show that the discussion of PLNC 1 in (Khoo, 2022) and others, be it positive or negative, should not be considered to reflect on **(ST)**. Even accepting **(ST)**, PLNC 1 is neither a particularly plausible nor the only candidate for the probability of left-nested conditionals. Furthermore, the performance of PLNC 2 as the immediate consequence of **(ST)** – when **(ST)** is interpreted in full generality – reflects positively on **(ST)**. For all its formally derivable triviality results, **(ST)** remains a remarkably adequate rule for the probability of indicative conditionals, even for the rather uncommon left-nested ones.

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The modal logic of verbal disagreement

ALEXANDER W. KOCUREK
Cornell University, USA
e-mail: awk78@cornell.edu

Many disputes seem difficult to resolve even after careful consideration of the evidence and reflection on the arguments. There are two views as to why such a dispute is so hard to resolve. According to **factualism**, the dispute in question is *factual* in that it arises from conflicting beliefs about the world. For factualists, the correct resolution of a dispute is a "factual matter": it is hard to resolve simply because it it is hard to establish the facts. According to **verbalism**, it is *verbal* in that it arises from conflicting interpretations of language. For verbalists, the correct resolution of a dispute is a "matter of interpretation": there are simply many admissible resolutions and we have yet to decide which to adopt.

Recently, I have been developing a philosophical theory of what makes a dispute verbal as opposed to factual (Kocurek, 2023). Here, I aim to do two things. First, I summarize this theory and its philosophical motivations. Second, I formalize the theory in a *modal* framework and present a sound and complete axiomatization for it.

Quick note of clarification: By 'verbal disputes', I refer to **wholly verbal disputes**, i.e., disputes that solely arise from differences in interpretation (cf. Plunkett and Sundell 2013; Thomasson 2017). Some use the term to refer to **merely verbal disputes**: disputes where speakers "talk past each other", i.e., misunderstand what the other side is saying (cf. Chalmers 2011; Jenkins 2014). For lack of space, I set aside merely verbal disputes and simply focus on what makes a dispute wholly verbal or wholly factual (or partly both).

On my preferred theory, a (wholly) verbal dispute is a kind of *practical* dispute over how to interpret words, rather than a factual dispute over what a word "really" means. To clarify, we follow Grice (1968) in distinguishing **speaker meaning**, i.e., what a speaker means by an expression in their idiolect ("S means m by e"), from **semantic meaning**, i.e., what an expression means in a language or linguistic community ("e means m in L"). While the semantic meaning of an expression may be determined by factors outside an individual speaker's control, it is generally acknowledged that speakers have at least some degree of control over what they speaker mean (Vermeulen 2018; Pinder 2021; Koch 2021). Though speakers often intend to align their speaker meaning with semantic meaning, they sometimes intend otherwise, especially if their aim is to *advocate* for their preferred meaning by "showing rather than telling" (Haslanger, 2000; Plunkett and Sundell, 2013; Thomasson, 2017).

Speaker meaning is therefore not solely determined by beliefs about semantic meaning. Rather, it is the output of what I call a **semantic plan**: a decision to associate certain meanings with expressions, either in general or for the purposes of conversation. While such plans are generally informed by one's beliefs (e.g., about the semantic meaning of a word) and desires (e.g., to have others use a term a certain way), the former is not easily reducible to the latter: semantic plans are *intentions* to use words in a certain fashion, rather than beliefs or desires. Like plans in general, a semantic plan can be *partial*, in that it does not fully specify the exact meaning of every expression. It can also be *conditional*, in that what meaning it assigns to a word may vary from world to world. Verbal disputes arise when the speakers' semantic plans recommend different interpretations of the disputed claim.

How do we tell when a dispute is verbal or factual? The answer is not so straightforward. While there are several "methods" for ascertaining the nature of a disagreement (e.g., Hirsch's (2009) method of translation or Chalmers's (2011) method of elimination), none is decisive.

Whether a dispute is verbal or factual turns on whether the two sides "mean the same thing" by the disputed claim (the factualist says yes, the verbalist says no). But it is not even clear this is a factual matter: on some views, what the speakers mean and believe—and thus, whether a dispute they have is verbal—is itself not solely a matter of fact, but at least partly a matter of interpretation (Davidson, 1974; Dennett, 1987; Stalnaker, 2014).

My goal here isn't to defend this view, but instead to show how to *formalize* it. To do this, we introduce a language $\phi ::= p \mid \neg\phi \mid (\phi \wedge \phi) \mid \mathsf{A}_k\,\phi \mid [\mathtt{f}]\,\phi \mid [\mathtt{v}]\,\phi$, where $\mathsf{A}_k\,\phi$ says speaker S_k accepts ϕ, $[\mathtt{f}]\,\phi$ says the speakers have a wholly factual disagreement over whether ϕ, and $[\mathtt{v}]\,\phi$ says they have a wholly verbal one. Throughout, we'll assume there are only two speakers, S_0 and S_1. We define $(\mathtt{d})\,\phi := (\mathsf{A}_0\,\phi \wedge \mathsf{A}_1\,\neg\phi) \vee (\mathsf{A}_0\,\neg\phi \wedge \mathsf{A}_1\,\phi)$ (this says the speakers have some kind of disagreement over ϕ). We'll also write $[\mathtt{x}]_k\,\phi$ for $\mathsf{A}_k\,\phi \wedge [\mathtt{x}]\,\phi$.

Models are tuples $M = \langle I, W, S_0, S_1, V\rangle$ where $I \neq \varnothing$ is a set of interpretations,[1] $W \neq \varnothing$ is a set of worlds, $S_k\colon (I \times W) \to \wp(I \times W)$ is a serial accessibility relation ($S_k(i,w) \neq \varnothing$), and $V(p) \subseteq I \times W$ is a valuation function. Intuitively, $\langle j, v\rangle \in S_k(i,w)$ iff S_k's beliefs and semantic plans at w according to i leave open that v is actual and leave open adopting j in v. We define $B_k(i,w) = \{v \in W \mid \exists j \in I\colon \langle j, v\rangle \in S_k(i,w)\}$ (i.e., the worlds left open by S_k's beliefs) and $I_k(i,w) = \{j \in I \mid \exists v \in W\colon \langle j, v\rangle \in S_k(i,w)\}$ (i.e., the interpretations left open by S_k's beliefs and semantic plans). Truth is evaluated at $\langle M, i, w\rangle$ triples. Here are the truth conditions for the modal operators (where $[\![\phi]\!] = \{\langle i, w\rangle \in I \times W \mid M, i, w \Vdash \phi\}$ and $[\![\phi]\!]^i = \{w \in W \mid M, i, w \Vdash \phi\}$):

$$
\begin{array}{lcl}
M, i, w \Vdash \mathsf{A}_k\,\phi & \Leftrightarrow & S_k(i,w) \subseteq [\![\phi]\!] \\
M, i, w \Vdash [\mathtt{f}]\,\phi & \Leftrightarrow & \exists k \in \{0,1\}\colon S_k(i,w) \subseteq [\![\phi]\!] \text{ and } S_{1-k}(i,w) \subseteq [\![\neg\phi]\!] \text{ and:} \\
 & & \text{(i) } \forall j \in I_{1-k}(i,w)\colon B_k(i,w) \subseteq [\![\phi]\!]^j \\
 & & \text{(ii) } \forall j \in I_k(i,w)\colon B_{1-k}(i,w) \subseteq [\![\neg\phi]\!]^j \\
M, i, w \Vdash [\mathtt{v}]\,\phi & \Leftrightarrow & \exists k \in \{0,1\}\colon S_k(i,w) \subseteq [\![\phi]\!] \text{ and } S_{1-k}(i,w) \subseteq [\![\neg\phi]\!] \text{ and:} \\
 & & \text{(i) } \forall j \in I_{1-k}(i,w)\colon B_k(i,w) \nsubseteq [\![\phi]\!]^j \\
 & & \text{(ii) } \forall j \in I_k(i,w)\colon B_{1-k}(i,w) \nsubseteq [\![\neg\phi]\!]^j\,.
\end{array}
$$

In words, a disagreement is wholly factual if each side would maintain their current position were they to adopt any interpretation left open by the other. A disagreement is wholly verbal if each side would no longer maintain their current position were they to adopt any interpretation left open by the other.[2] Consequence is defined as truth-preservation over $\langle M, i, w\rangle$ triples.

Figure 1 illustrates different kinds of "first-order" disputes in this framework. In these cases, the speakers involved agree over the status of their dispute. Figure 2 illustrates examples where the speakers also have a "metadispute" over the status of their first-order dispute. In the left model, the speakers agree that their metadispute is factual ($[\mathtt{f}]\,[\mathtt{f}]\,p$, $[\mathtt{f}]\,[\mathtt{v}]\,p$). Thus, they agree it is an entirely factual matter whether they "mean the same thing" by p. In the right model, one speaker holds their metadispute is verbal ($[\mathtt{v}]\,[\mathtt{f}]\,p$, $[\mathtt{v}]\,[\mathtt{v}]\,p$). That is, they maintain it is a matter of interpretation whether two sides "mean the same thing" by p. In fact, they also hold their *meta*metadispute is verbal (e.g., $[\mathtt{v}]\,[\mathtt{v}]\,[\mathtt{v}]\,p$)—and so on for all orders.

Observe that $[\mathtt{f}]$ and $[\mathtt{v}]$ are not normal modal operators (neither validates the K axiom or necessitation) and exhibit unusual logical properties (e.g., they're closed under negation). This raises the question: are these operators well-behaved enough to be axiomatized? The answer is

[1]Interpretations are treated as primitive points. The "interpretation" of p according to i is specified by V.

[2]Note: a disagreement can be partly factual and partly verbal, i.e., $(\mathtt{d})\,\phi \wedge \neg\,[\mathtt{f}]\,\phi \wedge \neg\,[\mathtt{v}]\,\phi$ is satisfiable.

yes. Figure 3 gives a sound and complete axiomatization for the semantics presented above.[3] I'll highlight three interesting logical properties this axiomatization reveals. First, $[\mathtt{f}]\,\phi$ and $[\mathtt{v}]\,\psi$ are inconsistent if either ϕ entails ψ or vice versa. Second, these operators are "convex": if ϕ entails ψ and ψ entails χ, then $[\mathtt{f}]\,\phi$ and $[\mathtt{f}]\,\chi$ entail $[\mathtt{f}]\,\psi$ (and likewise for $[\mathtt{v}]$). Third, the logical properties of $[\mathtt{f}]$ and $[\mathtt{v}]$ are not the same: e.g., $[\mathtt{f}]_k\,\phi$ and $[\mathtt{f}]_k\,\psi$ entail $[\mathtt{f}]_k(\phi \wedge \psi)$, yet $[\mathtt{v}]_k\,\phi$ and $[\mathtt{v}]_k\,\psi$ (even with $[\mathtt{v}]_k(\phi \vee \psi)$) don't entail $[\mathtt{v}]_k(\phi \wedge \psi)$.

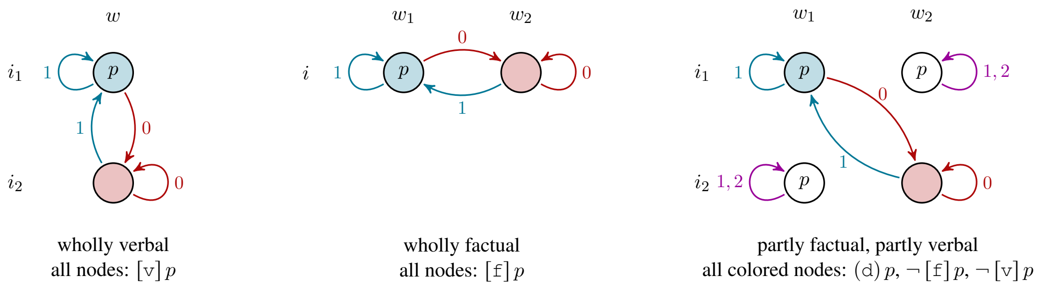

Figure 1: Models of different kinds of disputes. Nodes represent interpretation-world pairs. Arrows represent the accessibility relation. Color on nodes is just for readability.

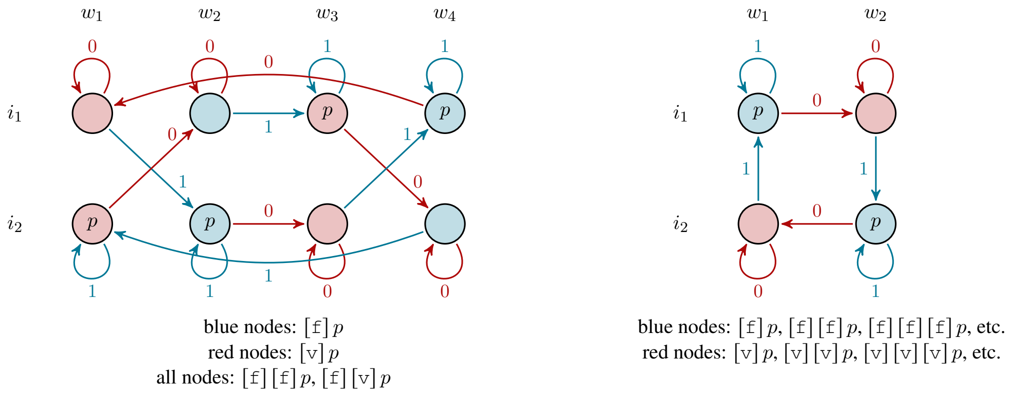

Figure 2: Models of "metadisputes" over the nature of a first-order dispute over p.

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[3]Proof omitted. Adding the usual axioms for A_k (e.g., 4 and 5) correspond to the usual properties on S_k (transitivity and euclideanness).

Axioms
$\vdash \phi$ where ϕ is an instance of a propositional tautology
$\vdash \mathsf{A}_k(\phi \to \psi) \to (\mathsf{A}_k\,\phi \to \mathsf{A}_k\,\psi)$
$\vdash \neg\,\mathsf{A}_k\,\bot$
$\vdash [\mathtt{f}]\,\phi \to (\mathtt{d})\,\phi$ and $\vdash [\mathtt{v}]\,\phi \to (\mathtt{d})\,\phi$
$\vdash [\mathtt{f}]\,\phi \to [\mathtt{f}]\,\neg\phi$ and $\vdash [\mathtt{v}]\,\phi \to [\mathtt{v}]\,\neg\phi$
$\vdash ([\mathtt{f}]_k\,\phi \wedge [\mathtt{f}]_k\,\psi) \to [\mathtt{f}]_k(\phi \wedge \psi)$ (reminder: $[\mathtt{f}]_k\,\phi = \mathsf{A}_k\,\phi \wedge [\mathtt{f}]\,\phi$)
Rules
if $\vdash \phi$ and $\vdash \phi \to \psi$, then $\vdash \psi$
if $\vdash \phi$, then $\vdash \mathsf{A}_k\,\phi$
if $\phi \dashv\vdash \psi$, then $[\mathtt{f}]\,\phi \dashv\vdash [\mathtt{f}]\,\psi$ and $[\mathtt{v}]\,\phi \dashv\vdash [\mathtt{v}]\,\psi$
if $\phi \vdash \psi$, then $[\mathtt{f}]\,\phi, [\mathtt{v}]\,\psi \vdash \bot$ and $[\mathtt{v}]\,\phi, [\mathtt{f}]\,\phi \vdash \bot$
if $\phi \vdash \psi \vdash \chi$, then $[\mathtt{f}]\,\phi, [\mathtt{f}]\,\chi \vdash [\mathtt{f}]\,\psi$ and $[\mathtt{v}]\,\phi, [\mathtt{v}]\,\chi \vdash [\mathtt{v}]\,\psi$
if $\alpha, \phi \vdash \psi$ and $\beta, \neg\phi \vdash \chi$, then $[\mathtt{f}]_k\,\alpha, [\mathtt{f}]_{1-k}\,\beta, [\mathtt{v}]_k\,\psi, [\mathtt{v}]_{1-k}\,\chi, (\mathtt{d})_k\,\phi \vdash [\mathtt{v}]_k\,\phi$

Figure 3: The modal logic of disagreement

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Versions and virtues of global validity

BAS KORTENBACH
Scuola Normale Superiore, Italy
e-mail: bas.kortenbach@sns.it

Recent developments in substructural logic have led to a burst of research on higher level inferences: metainferences (inferences from premise inferences to conclusion inferences), metametainferences (inferences from metainferences to metainferences), and so on. A primary catalyst for this has been the discovery in (Pailos, 2020) and (Barrio et al., 2020) that, for each inferential level, there exists a logic which coincides with classical logic precisely up to that level, but not beyond.

It is well known that upon ascending to the level of metainferences, one is faced with the choice between defining their validity criterion globally, as preservation of validity, or locally, as preservation of satisfaction per model (Humberstone, 1996). Whilst the global option is apparently natural, criticism has been mounting. Prominent allegations include that global validity is too weak a criterion (Barrio et al., 2020; Da Railos, n.d.; Dicher & Paoli, 2020), insufficiently analogous to validity for regular inferences (Barrio et al., 2020, 2021; Da Railos, n.d.), and fails to be closed under uniform substitution (Golan, 2021). Accordingly, the literature on metainferences in substructural logic has tended to focus on the local definition. In particular, studies considering inferences of arbitrary finite or transfinite levels typically rely exclusively on the higher-level generalization of local validity (Barrio et al., 2020; Ferguson & Ram?írez-Cámara, 2022; Fitting, 2021; McAllister, 2022; Pailos, 2020; Porter, 2022; Ripley, 2021; Scambler, 2020a,b). Defenses of global validity are hard to come by (Teijeiro's (2021) being the notable exception), and little to no work has gone into understanding how to apply it to inferences above the metalevel.

Our first purpose is to explore how global validity can be generalized to higher levels. We will see how from the metametainferential level onward, global validity itself splits into two notions. This happens because global validity admits two different definitions—as preservation of validity, or preservation of satisfaction-at-every-model—which are equivalent for the metalevel. The former can be generalized upwards as either preservation of local validity (the option we dub global-local validity), or non-equivalently as preservation of global validity itself (global-global validity). The global-local alternative retains higher-level equivalence with generalized preservation of satisfaction-at-every-model. We discuss how the new distinction interacts with a known (Da R al., 2020) bifurcation that occurs when lifting global validity from single- to multiple-conclusion inferences. We furthermore generalize Teijero's (2021) proof, that basic local and global validity collapse given certain general circumstances, to show that in these conditions both global versions are at least as strong as local validity on all inferential levels. The other direction of entailment, however, is maintained for global-local but not global-global validity. These results put pressure on the argument that the local criterion is preferable in virtue of being stronger.

This relates to our second purpose: to urge a reconsideration of global validity's viability. We will raise lines of defense against each of the main objections. While we naturally consider the allegations with respect to our generalized global notions, the responses offered also apply to the basic metalevel version. It is furthermore observed that global-global validity has a distinct advantage over both the local and global-local notions in being extensionally characterizable.

We conclude that the case against global validity is at least far less clear than is commonly suggested, and in particular that, depending on one's purposes, global-global can emerge as an alternative far superior to local validity.

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Aristotle's syllogistic as an arithmetical theory

LADISLAV KVASZ
Institute of Philosophy, Czech Academy of Sciences, Czech Republic
e-mail: kvasz@flu.cas.cz

There is a tension between Aristotelian logic and Ancient geometry. On the one hand, "*it is commonly supposed that Aristotle learned his use of letters from the geometers*" (Barnes 2007, p. 335). On the other hand, "*the codification of elementary mathematics by Euclid and the rich development of Greek mathematics in the third century are independent of logical theory*" (Mueller, 1974, p. 66). The aim of the present paper is to explain this tension. As observed already by Gallen, mathematical proofs contain *relational syllogisms* that fall outside the Aristotelian framework. Thus, we have to explain, why Aristotle's logic did not contain relational syllogisms and why is this kind of arguments central in mathematical proofs. To understand this question, we will turn to Frege's interpretation of the transition from the Aristotelian syllogistic logic to his version of mathematical logic in the paper *Funktion und Begriff* (Frege 1891, p. 30; English translation p. 40):

> If we look back from here over the development of arithmetic, we discern an advance from level to level. At first people did calculations with individual numbers 1, 3, etc. and $2 + 3 = 5$, $2 \cdot 3 = 6$ are theorems of this sort. Then they went on to more general laws that hold good for all numbers. What corresponds to this in symbolism is the transition to the literal notation. A theorem of this sort is $(a + b) \cdot c = a \cdot c + b \cdot c$. At this stage they had got to the point of dealing with individual functions; but were not yet using the word, in its mathematical sense, and had not yet formed the conception of what it now stands for. The next higher level was the recognition of general laws about functions, accompanied by the coinage of the technical term 'function'. What corresponds to this in symbolism is the introduction of letters like f, F, to indicate functions indefinitely. ... Now at this point people had particular second-level functions, but lacked the conception of what we have called second-level functions. By forming that, we make the next step forward.

The next step forward in the last sentence was of course Frege's own contribution.

For a long time, I considered this interpretation faithful and understood Frege's transition from Aristotelian logic to Begriffsschrift as a change in mathematical symbolism, that is, as the introduction of second level functions (the values of which are truth values). Nevertheless, this view is, as I would like to show in this paper, wrong. The transition from Aristotelian to Fregean logic was a much deeper change than an introduction of some new mathematical objects (i.e. second level functions). I would like to explain this change as *idealization*, i.e. as the change of the very character of language that we use in the construction of our idealized models (of reality, including thought, concepts or arguments).

The aim of the paper is thus to provide a new interpretation of the transition from Aristotle's logic to Frege's logic. I will try to show that Aristotle's logic can be considered *arithmetical* (in a technical sense of the word arithmetical), while Frege's logic is mathematical (in the same sense of the word mathematical). In order to introduce the adjectives 'arithmetical' and 'mathematical' in this technical sense, we must realize that language contributes to the formulation of our theories by connecting what would otherwise remain unconnected. Therefore, I shall speak of the synthetic role of language, and propose to distinguish three kinds of linguistic synthesis.

The first kind of synthesis I propose to call ***relational synthesis***, and I mean by it the ability of a language to bring different aspects of phenomena into relationships with each other. For example, Kepler's third law relates the third power a^3 of the length a of the main axis of a planet's orbit with the square T^2 of the time T of the planet's orbit around the Sun. This relation is phenomenally inaccessible because we cannot perceive or imagine the square of time. However, the language of algebra allows us to construct a quantity T^2 from the observed quantity T and relate it to a^3. Thus, the *language* of algebra allows the two aspects of the planet's motion that we can observe—a and T—to be put into relation to each other. I propose to call this ability of language its relational synthesis. In addition to relational synthesis, I propose to introduce ***compositional synthesis***, by which I mean the ability of language to construct representations of complex systems from elementary parts. Thus, for example, Newtonian mechanics allows us to represent the simultaneous motion of multiple bodies, whereas Aristotelian physics was only able to describe the motion of a single body. Aristotelian physics could not unite the description of motion of different bodies into a dynamical system. I propose to express this difference in the description of motion by saying that the language of Newtonian physics has (unlike that of Aristotelian physics) compositional synthesis. As a third kind of synthesis I propose to introduce ***deductive synthesis***, and I mean by it the ability of language to infer consequences that follow from representations of reality. The language of Newtonian mechanics allows us to infer the future state of a dynamical system from the knowledge of its present state and the forces at work.

Elementary arithmetic (i.e. simple reckoning) and mathematics (e.g., Euclidean geometry) have different relational, compositional, and deductive synthesis. *The relational synthesis* of arithmetic allows one to put the numbers obtained by counting into mutual relations of being smaller, equal, or larger. However, two numbers cannot be similar to each other nor can one number be perpendicular to the other. Thus, the language of arithmetic has a limited relational synthesis compared with geometry, where we have similarity of figures and two lines can be in different positions relative to each other. *The compositional synthesis* of the language of arithmetic is trivial: a number is a set of units, and a unit enters a number only by its presence. In contrast, in Euclidean geometry lines and circles enter into the construction of a composite geometric figure not only by their presence, but in many different relations to the rest of the figure. *The deductive synthesis* of the language of arithmetic is given by the rules of arithmetic, while in geometry it is given by the axioms that legitimize the steps of deductive inference. I will argue that *the relational, compositional, and deductive synthesis of Aristotelian logic* are similar to those of arithmetic, while the relational, compositional and deductive synthesis of Fregean logic are similar to those of mathematics.

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On many-valued neighborhood semantics for quantified modal logics

CHUN-YU LIN
Institute of Computer Science, the Czech Academy of Science, Czech Republic
e-mail: lin@cs.cas.cz

NICHOLAS FERENZ
Institute of Computer Science, the Czech Academy of Science, Czech Republic
e-mail: ferenz@cs.cas.cz

The algebraic Cintula-Noguera [3] and the frame-based Mares-Goldblatt [4] interpretations of quantifiers share the feature that (partially but ultimately) *non-complete* algebras are used to model the universal and existential quantifiers. The latter has been further shown to play well with modalities (e.g., see [4]). These formalizations capture the inferential essence of quantifiers and are philosophically well-motivated. We aim to generalize these approaches to construct neighborhood semantics (see, e.g., [6]) for first-order modal extensions of many valued logics. In particular, we aim to combine and extend [2] and [3] to first-order modal logics. This framework encompasses many mathematical fuzzy logics and many-valued logics. In [7], the researchers proposed a standard Łukasiewicz crisp Kripkean semantics for quantified modal logic. The complete axiomatization, nevertheless, has not been discussed. In this paper, we prove the soundness and completeness theorem for quantified modal logic with respect to constant domain many-valued neighborhood semantics.

The language $\mathscr{L}_1^{\Box}$ in this paper consists of $\wedge$, $\&$, $\to$, $\bot$, $\forall$, $\exists$ and the modal operator $\Box$ as our logical symbols, *Var* as a countable set of variables, *Con* as a set of object constants, and *Pred* as a set of predicate symbols. The formulas are constructed recursively as usual using predicates, logical connectives, quantifiers, and modality. We use $\mathbb{A}$ and A to denote MTL-chains and the domain of MTL-chains respectively.

Definition 1 *A constant domain $N(\mathbb{A})$-frame is a tuple $\langle W, N^{\Box}, D\rangle$ where W is a nonempty set of possible worlds, $N^{\Box} : W \to A^{A^W}$ is a neighborhood function, and D is a nonempty set. A constant domain $N(\mathbb{A})$-model is a 5-tuple $\langle W, N^{\Box}, D, I, C\rangle$ where $\langle W, N^{\Box}, D\rangle$ is a constant domain $N(\mathbb{A})$-frame, I is a predicate interpretation: for any n-ary predicate symbol $P \in Pred$ and any state $w \in W$, $I(P,w) : D^n \to A$, and C is a interpretation : for any object constants $c \in Con$, $C(c) \in D$.*

Definition 2 *An assignment $v : Var \to D$ is a function assigning elements of domain D to variables. An x-variant of an assignment v is an assignment v' such that for all $y \in Var$ with $y \neq x$, $v(y) = v'(y)$. We use $v \sim_x v'$ to denote that v' is an x-variant of v.*

Definition 3 *Suppose that $\mathbf{M} = \langle W, N^{\Box}, D, I, C\rangle$ is a constant domain $N(\mathbb{A})$-model and v is an assignment. For $\varphi \in \mathscr{L}_1^{\Box}$, we can define the evaluation $V^{\mathbf{M}}$ at a state w with respect to v by induction on the complexity of φ as follows:*

- $V^{\mathbf{M}}(w,v)(\bot) = 0$, $V^{\mathbf{M}}(w,v)(t) = \begin{cases} v(x) & \text{, if } t \text{ is a variable } x \\ C(c) & \text{, if } t \text{ is a constant } c \end{cases}$
- $V^{\mathbf{M}}(w,v)(P(t_1,\ldots,t_n)) := I(P,w)(V^{\mathbf{M}}(w,v)(t_0),\ldots,V^{\mathbf{M}}(w,v)(t_{n-1}))$,

- $V^{\mathbf{M}}(w,v)(\varphi \star \psi) = V^{\mathbf{M}}(\varphi) *^{\mathbb{A}} V^{\mathbf{M}}(\psi)$ *for* $\star, * \in \{\wedge, \rightarrow, \&\}$,
- $V^{\mathbf{M}}(w,v)(\Box\varphi) = N^{\Box}(w)(\|\varphi\|_{\mathbf{M},v})$, *where* $\|\varphi\|_{\mathbf{M},v}$ *is a function from* W *to* $\mathbb{A}$ *such that* $\|\varphi\|_{\mathbf{M},v}(w) = V^{\mathbf{M}}(w,v)(\varphi)$,
- $V^{\mathbf{M}}(w,v)(\forall x\varphi(x)) = \inf\{V^{\mathbf{M}}(w,v')(\varphi(x)) | v \sim_x v'\}$,
- $V^{\mathbf{M}}(w,v)(\exists x\varphi(x)) = \sup\{V^{\mathbf{M}}(w,v')(\varphi(x)) | v \sim_x v'\}$.

Since supremum and infimum might not exist, we consider safe models as usual. We use $\mathbf{L}_1^{\Box}$ to denote the standard Hilbert style proof system of many-valued predicate modal logic with the following axioms and rules :

1. Axioms of MTL logic and the five axioms of $(\forall 1)$ to $(\forall 3)$ in [5],
2. $\frac{\varphi\leftrightarrow\psi}{\Box\varphi\leftrightarrow\Box\psi}$ (RE), $\frac{\varphi\rightarrow\psi}{\varphi\rightarrow(\forall x)\psi(x)}$ (Gen) where x is not free in ψ, and $\frac{\varphi \quad \varphi\rightarrow\psi}{\psi}$ (MP).

The semantic consequence relation $\models_{\mathbf{N}(\mathbb{A})}$ with respect to a class $\mathbf{N}(\mathbb{A})$ of constant domain $N(\mathbb{A})$-frames is defined similarly as in [1]. Since the axiomatization of the propositional core is for the whole MTL-chains, therefore, we use $\models_{\mathbf{N}}$ to denote $\mathbb{A}$ running freely among all MTL-chains as in [5]. The provability relation $\vdash_{\mathbf{L}_1^{\Box}}$ between $\mathscr{L}_1^{\Box}$ theory T and formula φ is defined similarly as in [5]. We also use the similar definition of a theory T to be linear doubly Henkin in [5]. Let Con^+ be a countable infinite set of new constant symbol. Suppose that $\mathscr{L}_1^{\Box'}$ is the extension from $\mathscr{L}_1^{\Box}$ by adding constants symbols from Con^+.

Theorem 1 *Let* T *be a* $\mathscr{L}_1^{\Box}$*-theory and* Φ *be a directed set of* $\mathscr{L}_1^{\Box}$*-formulas such that* $T \nvdash \Phi$. *Then there is a linear* $\mathscr{L}_1^{\Box'}$*- doubly Henkin theory* $\hat{T}$ *such that* $\mathscr{L}_1^{\Box} \subseteq \mathscr{L}_1^{\Box'}$, $T \subseteq \hat{T}$ *and* $\hat{T} \nvdash \Phi$.

Definition 4 *The canonical model* $\mathbf{M}^* = \langle W^*, D^*, I^*, N^{*,\Box}, C^*\rangle$ *for some linear* $\mathscr{L}_1^{\Box}$ *Henkin theory* T *is constructed as follows:*

$$W^* = \{w : \mathscr{L}_1^{\Box} \rightarrow \mathbb{A} \mid w\text{:non-modal homomorphism with } w[T] \subseteq \{1\}\},$$

$D^* = Con^+ \cup Con$, $I^*(P,w)(t_1,\ldots,t_n) := w(P(t_1,\ldots,t_n))$ *for each* $w \in W^*$,

$$N^{*,\Box}(w)(|\varphi|_{\mathbf{M}}) := w(\Box\varphi)$$

where $|\varphi|_{\mathbf{M}^*} : W^* \rightarrow \mathbb{A}$ *with* $|\varphi|_{\mathbf{M}^*}(w) = w(\varphi)$. *The canonical assignment* $v^* : Var \rightarrow D^*$ *is defined as the identity map.*

One can show the following Truth-Lemma: that is, for each $w \in W^*$ and formula $\varphi \in \mathscr{L}_1^{\Box}$, $w(\varphi) = V^{\mathbf{M}^*}(w,v^*)(\varphi)$. Using Theorem 1 and the Truth-Lemma, we can get a proof of soundness and completeness theorem.

Theorem 2 *The class of all first-order constant domain* $N(\mathbb{A})$*-frames for any MTL-chains* $\mathbb{A}$ *is sound and strongly complete for* $\mathbf{L}_1^{\Box}$. *That is, for any theory* T *and formula* φ *of* $\mathscr{L}_1^{\Box}$,

$$T \vdash_{\mathbf{L}_1^{\Box}} \varphi \text{ iff } T \models_{\mathbf{N}} \varphi.$$

We then demonstrate how to generalize the frame-based Mares-Goldblatt interpretations of quantifiers toward MTL-chains and show that there is a correspondence between satisfaction of algebraic Cintula-Noguera and the frame-based Mares-Goldblatt semantics under an additional assumption.

The main upshots of our results consist of (1) a generalization of neighborhood semantics for predicate modal logic toward fuzzy setting, (2) a demonstration that the Cintula-Carles & Mares-Goldblatt quantifiers suitably interact with modalities in additional contexts, and (3) a basis for novel, more powerful (philosophical) interpretations of quantifiers in modal fuzzy logics.

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On Carnap categoricity for transitive modal logics

TADEUSZ LITAK
Informatik 8, Friedrich-Alexander-Universität Erlangen-Nürnberg, Germany
e-mail: tadeusz.litak@fau.de

Carnap [1943] argued that apart from soundness and completeness, one should consider *categoricity* as a fundamental criterion for relating syntax and semantics of a logical system. The importance and specific interpretations of this criterion have been long debated by, e.g., Church [1944], Smiley [1996], Rumfitt [2000], Raatikainen [2008], Murzi and Hjortland [2009], Garson [2013], or Hjortland [2014]. Bonnay and Westerståhl [2016, 2021] argued that in the modal setting it boils down to the question whether Kripkean semantics is forced upon us by a specific understanding of modal logic as the logic of possible worlds. Thus, the challenge is to find out when a normal modal logic Λ satisfies the following condition:

Kripkeanity every neighbourhood frame sound for Λ (i.e., validating all theorems of Λ) is *augmented* in the sense of Chellas [1980].

Recall that being *augmented* means that for any state/world x, its neighbourhood system $\mathsf{n}(x)$ is closed under arbitrary intersections; in other words, $\bigcap \mathsf{n}(x)$ is the smallest set in $\mathsf{n}(x)$. We can turn such a neighbourhood frame into an equivalent Kripke frame by taking $\bigcap \mathsf{n}(x)$ to be the set of successors of x.

This work provides a ZFC-based solution to a conjecture originally posed by Westerståhl and Holliday: that a normal extension of S4 is "Kripkean" in the above sense if and only if it is an extension of S5 (whose Kripkeanity can be established by standard algebraic techniques; cf., e.g., Holliday and Litak 2019, § 3 & § 7 for a short summary intended for a broader audience). Furthermore, the result is strengthened and generalized to show that a normal extension of K4 is Kripkean if and only if it is an extension of K4B. The main technical challenge of such a generalization lies in moving from *splittings* to *join-splittings*. Both concepts are routinely used in technical modal logic (the author was specifically inspired by the use of these concepts in Bezhanishvili and Harding 2007 and Blok 1978), yet do not appear to be well-known by philosophical logicians.

The fact that S5 is a splitting of the lattice of extensions of S4 has been established by Maksimova [1975]. In plain terms, this means that if a logic over S4 does not extend S5, all its theorems must be sound wrt the two-element linear order, which we may denote as $\mathsf{l}^{\circ}_{\circ}$. Thus, in order to show that a non-extension of S5 cannot be Kripkean, it is enough to display a non-augmented neighbourhood frame modally equivalent to $\mathsf{l}^{\circ}_{\circ}$. This is done by replacing its single leaf by an arbitrarily chosen infinite set X, fixing a non-principal ultrafilter Θ on X and postulating that neighbourhoods of the root contain a set from Θ (and the root itself).

By contrast, the logic K4B of transitive symmetric frames is not a splitting of the lattice of extensions of K4. However, using standard results such as those of Segerberg [1971] summarized in Chapter 8 of Chagrov and Zakharyaschev 1997, one can show that any extension of K4 not containing K4B must be sound wrt at least one of the following six frames: the four not-necessarily-reflexive variants of the two-element order ($\mathsf{l}^{\circ}_{\circ}, \mathsf{l}^{\circ}_{\bullet}, \mathsf{l}^{\bullet}_{\circ}, \mathsf{l}^{\bullet}_{\bullet}$), and the two variants of the three-element fork frame having both an irreflexive leaf and a reflexive leaf. Each of these frames allows a suitable variant of the S4 construction.

One can analogously reformulate the notion of Kripkeanity for superintuitionistic propositional logics (often called *intermediate logics*). A corollary of the above results is that the classical propositional calculus is the only Kripkean extension of intuitionism.

The above constructions involve non-principal (free) ultrafilters, whose existence over arbitrary infinite sets is guaranteed by the Boolean Prime Ideal Theorem (BPI). The use of BPI lies outside the scope of what Schechter [1997, § 14.76, p. 404] calls *quasiconstructive mathematics*, and Garnir [1974]—*agnostic mathematics*. Replacing an ultrafilter by, e.g., the *Fréchet filter* of cofinite subsets would refute formulas valid in certain posets, such as the axiom of linearity $\Box(\Box p \to q) \vee \Box(\Box q \to p)$. Actually, for numerous (possibly all) transitive logics of finite width not extending K4B, the failure of "Kripkeanity" is equivalent to a weak consequence of BPI, consistent even with set-theoretic principles contradicting BPI such as the *Axiom of Determinacy* (AD) of Mycielski and Steinhaus (see, e.g., Kanamori 2008, Chapter 6 or Litak 2018 for more information). The principle in question is named WUF(?) in Herrlich 2006 and states that there exists a free ultrafilter on *some* set.

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Infinitesimal credences without the axiom of choice

GUILLAUME MASSAS
University of California, Berkeley, USA
e-mail: gmassas@berkeley.edu

Non-Archimedean Probability theory (NAP) has recently been developed (Benci et al. (2013, 2018)) as a way to address limitations of classical Kolmogorov probability theory with respect to fair lotteries on infinite sets. While there can be no uniform countably (or evenly finitely) additive probability function assigning a non-zero probability to every non-empty subset of $\mathbb{N}$ when its range is the real interval $[0,1]$, NAP avoids this issue by defining the codomain of a probability function as a non-Archimedean field determined by the sample space under consideration.

Formally, given a sample space Ω, the codomain of a NAP function μ defined on $\mathcal{P}(\Omega)$ is obtained by taking first the ring $\mathcal{R}_\Omega$ of functions from $\mathcal{P}_{Fin}(\Omega)$ to $\mathbb{R}$, where $\mathcal{P}_{Fin}(\Omega) = \{A \subseteq \Omega \mid |A| < \omega\}$. In order to turn $\mathcal{R}_\Omega$ into a field, one then quotients it by a maximal fine ideal, i.e., a maximal ideal I containing all functions f that "eventually vanish", i.e. are such that there is some $A \in \mathcal{P}_{Fin}(\Omega)$ such that $f(B) = 0$ for any $B \supseteq A$. In the case of a fair lottery on Ω, one can then define a probability function $\mu : \mathcal{P}(\Omega) \to \mathcal{R}_\Omega/I$ by letting $\mu(A)$ for any $A \in \Omega$ be the equivalence class $[\phi_A]^I$ of the function $\phi_A : B \mapsto \frac{|A\cap B|}{|B|}$. Intuitively, the probability of an arbitary subset A of Ω is approximated by the conditional probability of A given a varying finite subset B. The result is a function that is defined on the whole powerset of Ω, is regular (i.e., $\mu(A) = 0$ implies $A = \emptyset$ for any $A \subseteq \Omega$) and satisfies a strong notion of additivity.

However, the definition of the codomain of a MAP function requires the use of a maximal fine ideal. In general, these are highly abstract objects whose existence can only be proved assuming a strong fragment of the Axiom of Choice. As a consequence, NAP has faced several criticisms. For one, the definition of the NAP function modelling a fair lottery on an infinite set Ω requires one to choose a specific fine ideal. As a consequence, there is no unique way for the NAP theorist to model such a fair lottery, which may lead one to suspect that the choice of a particular NAP function is always arbitrary or fails to represent a genuinely fair lottery. Moreover, the non-constructive nature of NAP functions arguably makes them an overly complex way of representing the credences of a rational agent regarding a fair lottery on an infinite set (Easwaran (2014)).

In this talk, I will discuss a small variation of NAP and argue that it answers many of the challenges raised against the non-Archimedean approach. The key idea is to replace the standard construction of the codomain of a NAP function with a system of quotients of rings. More precisely, given a sample space Ω, one considers the set of all quotient rings $\mathcal{R}_\Omega/J$, where J is an ideal containing all functions that eventually vanish. This set can then be partially ordered by reverse inclusion on the corresponding ideals (i.e., by letting $\mathcal{R}_\Omega/J \leq \mathcal{R}_\Omega/K$ iff $J \supseteq K$). Using an interpretation of the first-order language of fields on this poset that is reminiscent of forcing in set theory, one obtains a structure that is not a Tarskian model, but rather a *possibility structure* (Holliday (2021); Massas (2022)) that satisfies all the first-order axioms of a field. I will show how the main intuition behind NAP can still be adequately formalized in this setting and that the resulting structure allows one to model a fair lottery on an infinite set via a "canonical" NAP function.

Moreover, the definition of this structure does not require the Axiom of Choice, even though proving some of its nicer properties requires a fragment of the Axiom of Choice known as the Axiom of Dependent Choices (DC). I will therefore argue that possibility structures offer an elegant way of modelling infinitesimal credences in a semi-constructive context, i.e., in $ZF + DC$, arguably a natural foundational setting for analysis and probability theory.

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On the relation between non-deterministic and deterministic semantics — a modal perspective

PAWEL PAWLOWSKI
CLPS, Ghent University, Belgium
e-mail: haptism89@gmail.com

MARTA FIORI-CARONES
Sobolev Institute of Mathematics Novosibirsk, Russia
e-mail: marta.fioricarones@outlook.it

Our research article aims to enhance the comprehension of the relationship between modal logic and non-deterministic semantics, which is a crucial tool in various areas of computer science and artificial intelligence. The popularity of non-deterministic semantics has increased recently, as evidenced by the rising number of publications. The primary distinction between deterministic and non-deterministic semantics is extensionality. Deterministic semantics is extensional, meaning that the interpretation of connectives is functional and assigns a unique value to a formula constructed by a given connective. In contrast, non-deterministic semantics is not extensional, as the interpretation of a connective assigns a non-empty set of values rather than a unique value. Our objectives are twofold.

In [1] it was observed that there exists a validity preserving translation between any deterministic three-valued semantics and **S5**. More specifically, given a language for propositional logic $\mathcal{L}$ there exists a function T from formulas in $\mathcal{L}$ to modal formulas such that $\Gamma \vDash_3 \varphi$ if and only if $T(\Gamma) \vDash_{\mathbf{S5}} T(\varphi)$, for any Γ set of formulas in $\mathcal{L}$ and φ formula in $\mathcal{L}$. This connection between propositional three-valued semantics and modal semantics was proved by Tamminga and Kooi in [2] and then generalized by Kubyshkina in [3]. In the latter paper it is proved that a validity preserving translation exists between deterministic four-valued semantics and universal neighbourhood models too.

The existence of a validity preserving translation between two semantics is an interesting fact, and it is indeed not a novelty in the literature, since it allows to compare the properties of two semantics, to prove their relative consistency, and so it may carry interesting philosophical consequences. For these reasons we wondered, on the one hand, whether the above mentioned results hold for k-valued deterministic semantics, for $k > 4$, and, on the other hand, whether these results hold for non-deterministic semantics as well. The properties of non-deterministic semantics are less known than those of the more common determinist semantics, thus the existence of a validity preserving translation to a certain modal semantics may offer a tool to study their features.

Secondly, we investigate the relationship between non-deterministic and deterministic semantics. It is well-known that each non-deterministic matrix corresponds to a deterministic matrix with the same set of tautologies. However, if the matrix's set of values is finite, this is not always the case. There are examples of logic with a non-deterministic matrix even though they do not have a finitely many-valued deterministic one. We establish the conditions under which it is possible to determinize a non-deterministic matrix.

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Revolutions, paradigms and research programmes in logic: the case of the Realism *vs* Constructivism debate

ANTONIO PICCOLOMINI D'ARAGONA
Institute of Philosophy, Czech Academy of Sciences, Czech Republic
e-mail: piccolominidaragona@libero.it

(Kuhn, 2012) and (Lakatos, 1999) have provided useful tools for the analysis of the development of science. Kuhn aimed at amending Popper's theory of science as a permanent revolution" (Popper, 2002), by highlighting that what we usually mean by science is, rather, a puzzle-solving activity within the context of a dominant paradigm. Normal science is anticipated and followed by revolutionary periods where competing views fight on the corpse of an old paradigm. A new paradigm rises for mainly sociological, but in any case mostly non-scientific reasons. Lakatos tried to amend Kuhn's picture in turn, by arguing that science develops in sequences of theories. Each sequence comes with an untouchable core and a protective belt of hypotheses. The status of health of a research programme can be evaluated through some heuristic rules, partly reminiscent of Popper.

Kuhn's and Lakatos' approaches have proved to be very fruitful in applications to empirical sciences. It is still much debated, though, whether they can be applied to formal sciences too. As witnessed by (Gillies, 1992b), a number of views has been put forward starting from mathematical fields such as geometry, calculus and set theory. More recent works, like (Bueno, 2007) and (Oliveri, 2006), have seemed to imply that a Kuhnian-Lakatosian reconstruction of (specific case-studies in) the history of mathematics is actually consistent.

The applicability of Kuhn's and Lakatos' theories to logic–when the latter is understood as a sub-field of mathematics–is however mostly unexplored. The issue is of course connected to that about whether we can speak of proper revolutions in logic. To my knowledge, only two sources can be mentioned here: (Gillies, 1992a) and (Cellucci, 2001). Gillies is a *discontuinist*, in that for him we can speak of a Fregean revolution, essentially because of the foundational role which Frege thought logic had to play in mathematics. Cellucci is a *continuist*: there is no substantial difference between the Aristotelian approach and the Fregean one, since both aim at justifying science (or a sub-field of it), and since Frege's logic respects the deductivist attitude born with Aristotle. Cellucci thus claims that a logical turn could only be given by the development of a heuristic attitude, eventually leading to a logic of discovery (whose roots are also to be found in Aristotle). Cellucci's and Gillies' views may however not be entirely incompatible, as one may claim that Frege provoked a *linguistic* revolution in logic–possibly along the lines of (Kvasz, 2008).

The topics illustrated so far will constitute the starting point of my talk, which will otherwise concern, not the general issue about whether the history of modern logic can be read as a whole in Kuhn's or Lakatos' terms, but that about whether this can be done for the specific case-study of the opposition between logical-mathematical realism and logical-mathematical constructivism. In particular, I will argue that a number of historical and conceptual reasons led to the establishment of a Kuhnian *realist paradigm* (RP), given by the combination of Model Theory (MT) and (axiomatic) Set Theory (ST). Against this, one can detect a Lakatosian *constructivist research programme* (CRP), whose semantic import is instantiated for example by Prawitz's Proof-Theoretic Semantics (PTS)–see e.g. (Prawitz, 1973)–and whose foundational import is instantiated for example by Martin-Löf's Intuitionistic Type Theory (ITT)–see e.g. (Martin-Löf, 1984).

RP was established as a solution to the crisis provoked by the discovery of semantic and set-theoretic paradoxes in earlier attempts at giving mathematics either logical or naïves set-theoretical foundations and, then, by Gödel's incompleteness results. This led to the abandonment of foundational stances along the lines of the two major schools of logicism and formalism (and partly also of the intuitionistic one) and to the adoption of some new dogmas", like the object-language/meta-language distinction, or the idea that mathematical entities and methods are reducible to their set-theoretic counterparts.

If this reconstruction is correct, two further issues arise. As nothing seems to impede that MP + ST could have constituted a non-realist setup, the first question is why and in what sense RP is realist". I will argue that this stems from the results that a realist approach allowed for with respect to the aims that logicians had around the 1930s. For the very same reason, though, RP should not be understood as an entirely *new* paradigm with respect to the one dominating at the time of the earlier foundational schools. Rather, it is an evolution of the logicist-formalist side of the latter–modulo the Cellucci-Gillies opposition.

This *cannot* be understood as a research programme in Lakatos' sense, for RP touched the cores of the earlier foundational projects, by replacing some of their principles with new ones. The Lakatosian reading seems to be more adequate in the case of PTS and ITT, which I understand as belonging to a sequence of approaches given by, roughly, a combination of intuitionism and formalism. I end by focusing on what I take to be the proper interpretation of CRP–within the context of a general notion of *logical research programme*.

First, CRP still constitutes a minor approach in the logical community. Second, it does not require a principles-replacement in the cores of the theories underlying it. Third, it shows great flexibility in the replacement of assumptions in what I take to be the protective belt of the programme. Fourth, it seems to involve an implicit heuristics, hinting at desirable results, or at dangerous" outcomes which require interventions on the protective belt.

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Particular propositions as divisions: Lichtenfels and an Austrian tradition in logic

MARCEL QUARFOOD
Umeå University, Sweden
e-mail: marcel.quarfood@umu.se

There have been several uses of division in the history of logic. Traditionally, the subdivision of a genus concept in mutually exclusive species concepts that together are exhaustive of the genus concept is called logical division. In Boole's algebra of logic, division was used as an algebraic operation, sometimes resulting in formulas lacking clear interpretation. In his classical *Symbolic Logic* (2nd edition 1894), John Venn analysed problems connected with Boole's division. In passing he notes some other uses of division among early attempts to apply symbolic methods to logic in the 18th and 19th centuries, among them "a peculiar fractional representation of particular propositions" employed by some authors (Venn 1894, 497). These logicians had "adopted the unfortunate device of indicating particular propositions by a fraction" (Venn 1894, 87n). They symbolised the particular proposition "Some S is P" as $1/S - P$ (or variants such as $1/S = P$, or $1/S < P$), to indicate that only a part of the subject term is dealt with. "But why S is placed in the *denominator* of the fraction passes mathematical comprehension" (Venn 1894, 87n). Venn mentions several authors who use this notation (all writing in German), and he tentatively traces the tradition back to a book by Anton Victorin that appeared in 1835 (Venn 1894, 495-497).

Venn doesn't spell out what the problem is with having S as denominator of the fraction, but presumably it is that as a particular proposition is to indicate that some of the individuals in the extension of S are P, this should rather be symbolised by placing S in the numerator. If e.g., a third of the cats are black, we might write $S/3 - P$. And perhaps $S/x - P$ (where $x \geq 1$ and doesn't exceed the number of individuals in S) for an indeterminate number of individuals being P.

In this paper I attempt to do three things: first to suggest a possible explanation for these authors' strange placement of the subject term in the fraction; secondly to give a new hypothesis of whom it originated from; and thirdly to speculate on whether the group of authors using this device constitutes a specific Austrian tradition in logic.

First, I tentatively suggest that this notational device is connected to the traditional doctrine of logical division. Venn's reaction against placing S in the denominator is, I think, motivated by an extensional approach to logic. If we instead assume the traditional account of logical division, a concept (genus) is subdivided in subconcepts (species), which reflects an intensional rather than an extensional view. The extension (*Umfang*) of a concept is taken primarily to consist of subordinated concepts (rather than of individuals). We can then think of $1/S - P$ as indicating that one of the species of which the genus S is composed is singled out and said to be a P. Such a species is itself a concept, and it is a part of the whole of the subject, which might make the notation a little less unnatural. Unfortunately, not much explanation of the notational device is forthcoming from the authors themselves, but Victorin (1835, 47 and 97), Zimmermann (1863, 47) and Jäger (1839, 33) give accounts that can be read in the way I suggest.

Secondly, I correct Venn's conjecture that Victorin (1835) invented this notational device. Venn notes Lichtenfels's (1842) use of it. But there is the earlier Lichtenfels (1833) where the same notation is found, predating the book Venn refers to. Lichtenfels as originator makes much more sense, since he wrote textbooks widely used in Austrian universities, whereas Victorin seems to have been a rather isolated figure. Finally, I note that almost all authors using this device (adding some to Venn's list) worked in the Austrian empire. They also shared some views, e.g., that logic should be separated from psychology and metaphysics; that logicians ought to take a closer look at language; and they introduce some amount of symbolic notation. This is reminiscent of typical traits of other schools of Austrian philosophy pointed out by R. Haller (1979). One might therefore try to group these authors in a common Austrian school of logic. But perhaps this is to go too far; while several of them were followers of F. H. Jacobi's religious philosophy, at least one of the authors, the influential Bohemian philosopher and educator G. A. Lindner, was an adherent of J. F. Herbart's even more strictly formal conception of logic. In any case, this group represents an interesting 19th century trend of importing mathematical symbols into logic, without still really taking the steps needed for a genuinely mathematical logic.

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Dialectical disposition expressivism and its intrinsic logic

LIONEL SHAPIRO
University of Connecticut, USA
e-mail: lionel.shapiro@uconn.edu

1 Introduction

It is a familiar idea that the behavior of logical operators can be illuminated in terms of their use in dialectical engagement (e.g. Lorenzen and Lorenz, 1978; Lance, 2001; Brandom, 2008; Price, 1990). Typically, specifying the dialectical role of operators has been understood as a way to provide semantic grounding for some relation of logical consequence (e.g. intuitionistic, relevant, or classical).

This paper concerns a more modest application of the dialectical approach, *dialectical disposition expressivism* (Shapiro, 2023). On that view, sentential connectives serve to let speakers convey certain dispositions with regard to dialogue. I argue that this explanation of the function of connectives should not be required to settle what is a logical consequence of what. Still, one may ask whether the view gives pragmatic significance to any formal consequence relation(s) definable in terms the concepts it employs. If so, such a relation might be deemed an "intrinsic logic" of dialectical disposition expressivism, even if that relation should not be identified as logical consequence.

To address this question, I present a natural way to interpret sequent rules in terms of dialectical dispositions. The interpretation exploits a parallel between clauses specifying the expressive function of connectives and some of the inference rules of bilateralist systems of natural deduction (Rumfitt, 2000). I then investigate which sequent rules can be justified on the interpretation. I argue that the system of justifiable rules yields a consequence relation weaker than the intersection of intuitionistic logic and FDE.

2 From expressive function to sequent rules

Dialectical disposition expressivism holds that logical connectives serve to express dispositions with respect to moves in a Brandomian "game of giving and asking for reasons" whose moves include both *asserting* and *rejecting* (the latter understood as expressing a disposition to challenge). As an example, take disjunction. In asserting $A \vee B$, a speaker expresses the following disposition they have with regard to that very assertion:

($\vee$-c) The speaker is prepared to acknowledge an interlocutor's pair of rejections of A and of B as a *challenge* to their assertion.

($\vee$-m) When an interlocutor has challenged their assertion, the speaker is prepared to adduce, as a way to *meet* the challenge, any assertion of A (likewise of B) which the speaker is prepared to make or defer to.

These clauses resemble, respectively, the negative and affirmative $\vee$-introduction rules of a bilateralist natural deduction system, given here in sequent format:

$$\frac{\Gamma \vdash -A \qquad \Gamma \vdash -B}{\Gamma \vdash -A \vee B}\,(-\vee\mathrm{R}) \qquad \frac{\Gamma \vdash +A\ [+B]}{\Gamma \vdash +A \vee B}\,(+\vee\mathrm{R})$$

We can interpret such rules in terms of ideal agents, understood as ones who exhibit all dispositions they express and recognize what dispositions their assertions would express.

- $\Gamma \vdash_a -C$ iff for some $\Gamma' \subseteq \Gamma$, agent a acknowledges this combination of speech acts by an interlocutor as *challenging* their assertion of C: asserting the $+$-signed members of Γ', while rejecting the $-$-signed members.

- $\Gamma \vdash_a +C$ iff for some $\Gamma' \subseteq \Gamma$, agent a is disposed to adduce this combination of their own speech acts as *meeting any challenge* to their assertion of C: asserting the $+$-signed members of Γ', while, for each $-$-signed member, asserting something a acknowledges as challenging its assertion.

- The rule with premises $\Gamma_i \vdash \phi_i$ and conclusion $\Delta \vdash \psi$ holds iff for any ideal agent a, if $\Gamma_i \vdash_a \phi_i$ for all i, then $\Delta \vdash_a \psi$.

In giving sequents with differently-signed succedents distinct readings, the proposal resembles the use of "dual" turnstiles for proof and refutation in Wansing (2017) and Ayhan (2021).

3 A weak logic

Which sequent rules can be justified under this dialectical interpretation? I start by considering a signed Gentzen system for FDE, corresponding to the full set of bilateralist connective rules for $\wedge$, $\vee$, and $\neg$. Those rules I argue *cannot be justified* include

$$\frac{\Gamma, -A \vdash \phi \qquad \Gamma, -B \vdash \phi}{\Gamma, -A \wedge B \vdash \phi}\,(-\wedge\mathrm{L}) \qquad\qquad \frac{\Gamma, +A \vdash \phi}{\Gamma, -\neg A \vdash \phi}\,(-\neg\mathrm{L})$$

Their omission ensures that derivable sequents composed of $+$-signed sentences are intuitionistically valid. But the rules that cannot be justified include the third left-introduction rule

$$\frac{\Gamma, -A\ [-B] \vdash \phi}{\Gamma, -A \vee B \vdash \phi}\,(-\vee\mathrm{L})$$

Omitting this rule as well further weakens the resulting consequence relation. In examining which rules are justifiable, I consider the non-connective "reversal" rules

$$\frac{\Gamma, +A \vdash +B}{\Gamma, -B \vdash -A} \qquad \frac{\Gamma, +A \vdash -B}{\Gamma, +B \vdash -A} \qquad \frac{\Gamma, -A \vdash +B}{\Gamma, -B \vdash +A} \qquad \frac{\Gamma, -A \vdash -B}{\Gamma, +B \vdash +A}$$

On the proposed interpretation, these rules link challenging with meeting challenges. I argue against each rule even where Γ is empty.

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A general schema for bilateral proof rules

RYAN SIMONELLI
University of Chicago, United States
e-mail: simonelli@uchicago.edu

Logical bilateralism of the sort proposed by Smiley (1996) and Rumfitt (2000) provides rules for both *assertions* and *denials* of formulas, prefixing each formula with a positive or negative force-marker to indicate the assertion or denial of that formula.[1] Formalisms of this sort have been used to provide proof-theoretically virtuous natural deduction systems along the lines of Gentzen's NK, and have been prominent in the development of proof-theoretic semantics for logic and even natural language. There are, however, two worries regarding such systems. First, such systems seem to proliferate rules, generally requiring twice as many rules as their unilateral counterparts.[2] Second, such systems seem to provide too much freedom, with a number of different sets of rules being put forward as definitive of the meanings of the classical connectives.[3] In this paper, I respond to both of these concerns, providing a single schema that yields the rules for all of the connectives and which has a reasonable claim to a privileged status among bilateral rule forms.

The main system in the context of which these rules are proposed is a single conclusion bilateral sequent calculus where each connective is given exactly two rules: a positive rule saying when one is committed to affirming a sentence with that main connective and a negative rule saying when one is committed to denying a sentence with that main connective. The sole axiom schema is that of Containment (or Contexted Reflexivity): $\Gamma, A \vdash A$ (where Γ and $\{A\}$ contain only signed atomics). The one substantive structural rule that is necessary for the sequent calculus to function is the generalized contraposition principle that Smiley (1996) dubs *Reversal*. Where A and B are signed formulas, and starring a signed formula yields the oppositely signed formula, the principle enables you to infer $\Gamma, B^* \vdash A^*$ from $\Gamma, A \vdash B$. Conceptually, this principle can be understood as formally encoding the symmetry of *contraeity* and *subcontraeity* relations. Its technical significance, however, is that it eliminates the need for left rules, since one can get a signed formula with a given connective on the left side of the turnstile by getting its opposite on the right and using Reversal.[4]

Now, the key innovation of Smiley/Rumfit style bilateral natural deduction systems are the rules for negation which codify the equivalence of denying some sentence and affirming its negation as well as the equivalence of affirming some sentence and denying its negation. To Rumfitt's positive and negative negation introduction rules, I add the following general schema for the binary connective rules, where $\circ$ is any binary connective, $\boldsymbol{a}$, $\boldsymbol{b}$, and $\boldsymbol{c}$ are positive or negative force-indicator signs, and starring a sign yields the opposite of that sign:

[1]Note, this contrasts with *interpreting* multiple conclusion sequent calculi like Gentzen's LK bilaterally, as has become popular in recent years following Restall (2005) and Ripley (2013).

[2]Rumfit's system, for instance, has twice as many rules as Gentzen's NK, and this is the norm. Smiley proposes paired down system with half the number of rules as Rumfitt's, but he does so only by leaving out negative rules for conjunction and positive rules for disjunction.

[3]For instance, different rules have been provided by Smiley (1996), Rumfitt (2000), Francez (2014), Kürbis (2016), and del Valle-Inclan and Schlöder (2023), among others, all meeting standard (unilateral) harmony constraints, with many of these authors proposing their preferred versions of bilateral harmony that their proposed rules meet.

[4]A similar idea is at play in Smiley's (1996) paired down natural deduction system in which only positive rules are given for conjunction and only negative rules for disjunction.

$$\frac{\Gamma \vdash \boldsymbol{a}\langle\varphi\rangle \quad \Gamma \vdash \boldsymbol{b}\langle\psi\rangle}{\Gamma \vdash \boldsymbol{c}\langle\varphi \circ \psi\rangle}\ \boldsymbol{c}_\circ \qquad\qquad \frac{\Gamma, \boldsymbol{a}\langle\varphi\rangle \vdash \boldsymbol{b}^*\langle\psi\rangle}{\Gamma \vdash \boldsymbol{c}\langle\varphi \circ \psi\rangle}\ \boldsymbol{c}^*{}_\circ$$

The first rule says that if Γ commits one to taking stance $\boldsymbol{a}$ to φ and Γ also commits one to taking stance $\boldsymbol{b}$ to ψ, then Γ commits one to taking stance $\boldsymbol{c}$ to $\varphi \circ \psi$. The second rule is can be understood as saying that if, relative to Γ, the stances $\boldsymbol{a}\langle\varphi\rangle$ and $\boldsymbol{b}\langle\psi\rangle$ are incompatible in that Γ along with $\boldsymbol{a}\langle\varphi\rangle$ commits one to taking the opposite stance of $\boldsymbol{b}$ ($\boldsymbol{b}^*$) to ψ, then Γ commits one to taking stance the opposite stance of $\boldsymbol{c}$ ($\boldsymbol{c}^*$) to $\varphi \circ \psi$. The main technical result is a general bilateral analogue to Cut Elmination that shows that the bilateral Coordination Principle (Smileian Reductio):

$$\frac{\Gamma, A \vdash B \quad \Delta, A \vdash B^*}{\Gamma, \Delta \vdash A^*}$$

is admissible with respect to rules of this form.[5] It follows that a logic L consisting in rules of this form is consistent in the sense that, if $\vdash_L A$, then $\nvdash_L A^*$.

All of the classical connectives can be defined with rules of this form simply by varying the signs assigned to $\boldsymbol{a}$, $\boldsymbol{b}$, and $\boldsymbol{c}$.[6] When the rules for given connective have been provided in accordance with this schema, the rules for its dual can be obtained simply by taking the opposite of all the signs. The instance of this general schema that is most immediately obvious are the rules for the conditional, which are are simply the introduction rules provided in Rumfit's (2000) natural deduction system. However, sequent rules for conjunction and disjunction following exactly the same schema can be given. For instance, we have the following rules for conjunction:[7]

$$\frac{\Gamma \vdash +\langle\varphi\rangle \quad \Gamma \vdash +\langle\psi\rangle}{\Gamma \vdash +\langle\varphi \wedge \psi\rangle}\ +_\wedge \qquad\qquad \frac{\Gamma, +\langle\varphi\rangle \vdash -\langle\psi\rangle}{\Gamma \vdash -\langle\varphi \wedge \psi\rangle}\ -_\wedge$$

The system consisting in these bilateral connective rules is a sound and complete system of classical logic. In fact, it's equivalent to Ketonen's (1944) multiple conclusion classical sequent calculus, which has several nice formal properties all of which are preserved in this system.[8] However, whereas Ketonen's system is essentially multiple conclusion, this bilateral system uses only single conclusion sequents, and thus provides a more immediately intuitive explication of the sense of the connectives.[9]

In addition to the standard classical connectives, the same general rule schema also yields rules the Sheffer Stroke and Pierce's Arrow.[10] Finally, though the above schema alone suffices for complete sequent calculi with one's choice of connectives, one can also supplement the schema with schematic left rules (obtained immediately through Reversal) for a more traditional sequent calculus. Alternately, one can supplement the schema with the following schematic elimination rules to yield a harmonious natural deduction system with one's choice of connectives:[11]

[5]The proof proceeds analogously to standard Cut Elimination for sequent calculi but at this higher level of generality.

[6]Precisely, any connective in which there is a distinguished row of the truth-table has rules of this form.

[7]Rules of this sort have been recently been suggested by del Valle-Inclan and Schlöder (2023).

[8]Proof of this claim is given by providing a translation procedure for mapping unilateral multi-succident sequents to equivalence classes (under Reversal and Permutation) of bilateral single-succident sequents and an induction on proof height that shows that each step in one system corresponds to a step in the other.

[9]See, for instance, Rumfit (2000, 2008) for a relevant criticism of multiple conclusion sequent calculi.

[10]The rules yeilded by the schema (which the reader should be able work out) are equivalent to the multiple conclusion sequent rules provided by Riser (1967) and Zach (2016).

[11]In the context of natural deduction system, one takes the Smileian Reductio as basic rather than Reversal, and the reversibility of the hypothetical introduction and corresponding elimiantion rules is derived (See Del Valle-Inclan and Schlöder (2023, 200n3) on this point.)

$$\frac{\Gamma \vdash \boldsymbol{c}\langle \varphi \circ \psi \rangle \quad \Gamma \vdash \boldsymbol{a}\langle \varphi \rangle}{\Gamma \vdash \boldsymbol{b}\langle \psi \rangle}\ \boldsymbol{c}{\circ_E} \qquad \frac{\Gamma \vdash \boldsymbol{c}^*\langle \varphi \circ \psi \rangle}{\Gamma \vdash \boldsymbol{a}\langle \varphi \rangle}\ \boldsymbol{c}^*{\circ_{E_L}} \qquad \frac{\Gamma \vdash \boldsymbol{c}^*\langle \varphi \circ \psi \rangle}{\Gamma \vdash \boldsymbol{b}^*\langle \psi \rangle}\ \boldsymbol{c}^*{\circ_{E_R}}$$

To this latter end, I also provide generalized harmony results for rules of this form, using the standard expansion/reduction procedure (c.f. Pfenning and Davies, 2001) for unilateral harmony and the procedure recently proposed by del Valle-Inclan and Schlöder (2023) for bilateral harmony.[12]

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[12] Bilateral harmony for the introduction rules is immediately secured by the main theorem.

The untyped lambda calculus and natural language semantics

WILL STAFFORD
University of Bristol, UK
e-mail: Will.Stafford@bristol.ac.uk

Fox and Lappin propose using Property Theory with Curry Types (PTCT) to generate semantic representations for natural language. They claim that there are several advantages to their approach: it allows for hyperintensionality, it has a rich theory of types, and it is of weaker formal power than several of the main alternatives. We will evaluate this last claim and argue that the inclusion of the untyped λ-calculus in their system means that they fail to produce a theory which is weaker than the alternatives. While Fox and Lappin's work has been recognised as a novel approach to hyperintensionality, there has, as of yet, been little evaluation of their project. We hope to have here contributed to the larger debate about hyperintensionality by offering an evaluation of the formal power of PTCT.

PTCT includes the untyped λ-calculus, a Curry typing system, and a first-order language (Lappin , 2013, p. 181). The Curry typing system allows PTCT to accommodate polymorphism, and the untyped λ-calculus allows it to be hyperintensional. A hyperintensional logic is one that allows terms that pick out the same reference can have different semantic values. Two terms in the untyped λ-calculus can denote objects encoding the same function in a model while not picking out the same object.

Fox and Lappin are very concerned with having a theory that has limited formal strength while still being expressive enough for natural language semantics. This is, in fact, the only one of their motivations to which they devote a whole chapter.

> If we are interested in building practical natural language systems, then it is appropriate to worry about the computational properties of a semantic theory. (Fox and Lappin , 2005, p. 153)

This motivation makes sense given that their work is in computational semantics. Computational semantics investigates effectively implementable formal analyses of meaning. Here, an effective implementation means that in principle we could implement a computational procedure which takes us from natural language to a formal analysis of the meaning and which allows us to reason with the results (Blackburn and Bos , 2005, p. iii). (Fox , 2010, p. 394) also claims that computational semantics requires that the behaviour of the semantic representations can be expressed independently of the model theory. This is because formalisations which do not rely on the model theory are more likely to be effectively implementable. Having a complete, recursively enumerable proof theory, for instance, would meet this requirement by allowing us to work only with the proof system.

(Fox and Lappin , 2005, p. 151) claim that a decidable system is preferable to a semi-decidable one, which in turn is preferable to an not semi-decidable system. This leads them to claim that we should prefer first-order logic over second-order logic. For, first-order theories can be decidable or semi-decidable, while the standard semantics makes second-order logic not semi-decidable. However, first-order logic by itself isn't sufficiently expressive for natural language semantics, so we must extend it while attempting to limit the formal power (Fox and Lappin , 2005, p. 152). Fox and Lappin claim that their system achieves this goal.

It is not clear that their argument succeeds in revealing a weakness with alternative proposals. After all, to counter any semantic arguments against theories with the syntax of a second-order theory we can appeal to the general fact that second-order logic with Henkin semantics can be translated into a first-order theory (Shapiro , 1991, pp. 75-6). The use of second-order quantifiers or higher order types does not tell us that a theory is not first-order in power. Fox and Lappin are aware of this (Fox and Lappin , 2005, pp. 151, 156, 159-160), but fail to offer a robust argument against the adoption of Henkin semantics. This is not point that can be raised against their claim, as it can also be shown that their system substantially more powerful than typed systems.

Rice's theorem states that there is no general and computable method of discovering whether, for a non-trivial property of partial computable functions, a particular function has that property (Soare , 1999, p. 21). It turns out that there is an analogous result for the untyped λ-calculus. This results states that given a set A of λ-terms, if A is closed under provable equality in the λ-calculus, then A is either trivial or not decidable (Barendregt , 1981, p. 144). Rice's theorem also means that there is no λ-term which encodes the characteristic function of any set of λ-terms closed under equality because the untyped λ-calculus is Turing complete (Fernández , 2009, p. 34).

One corollary of Rice's theorem is that equality between λ-terms is not decidable in the untyped system. If M and N are λ-terms then deciding whether $M = N$ is provable is the same as deciding if $N \in \{X \mid \lambda \vdash X = M\}$. And $\{X \mid \lambda \vdash X = M\}$ is a set of terms closed under equality. This is in contrast with the simply typed λ-calculus and certain extensions of it. We say that a λ-term is in *normal form* if we cannot apply β-reduction to it (Girard , 1989, p. 18). A system is *strongly normalizing* if every sequence of applications of β reduction eventually reaches a normal form. A system is *confluent* if for any term t and any two terms t_1 and t_2 produced by applying β-reduction to t there is a third term t_3 which results from applying β-reduction possibly multiple times to t_1 and to t_2 (Baader and Nipkow , 1998, p. 3). The simply typed λ-calculus is strongly normalising and confluent (Girard , 1989, p. 45), as is an extension of it which allows for polymorphic types (Girard , 1989, p. 118). This means there is a procedure for deciding if $M = N$ is provable in these systems. The untyped λ-calculus is not normalising because there are terms to which we can always apply one of the rules of the calculus. For example, we can always apply β-reduction to $\lambda.x(xx)\lambda x.(xx)$ so it does not have a normal form.

But this means that on PTCT's view of intensions it is not obvious when two terms have the same intension. This is an odd result given what intensions were supposed to capture. While it is true that it is difficult to say whether two terms have the same intension, this appears to be due to philosophical disagreements about what is necessary for two terms to be intensionally distinct. It would seem odd to claim that this problem was in fact only semi-decidable, as most people think that deciding whether something has the same intension is an intuitive ability native speakers possess. But this is the result we are left with on Fox and Lappin's picture.

Fox and Lappin offer a well-motivated and innovative attempt to move away from the tradition of Montague grammars. However, they try to distinguish their work from work in that tradition by arguing that their system is first-order and so weaker computationally. We saw that this does not hold up for two reasons. Firstly, second-order theories can be seen as essentially first-order by moving to Henkin semantics. This means that there is no important difference between Fox and Lappin's proposal and theories that use higher-order typed systems. Secondly, the untyped λ-calculus which forms the basis of Fox and Lappin's system is computationally a very powerful theory, as it is Turing complete. This means that many things are not decidable in this system that are decidable in the typed λ-calculi. These points together show that Fox and Lappin have failed to produce a theory which is computationally more tractable than theories in the tradition of Montague grammars.

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Relevant logics and theories of topic

ANDREW TEDDER
Ruhr University Bochum, Germany
e-mail: ajtedder.at@gmail.com

Relevant logics are (usually) distinguished by their satisfaction of Belnap's *variable sharing property* (VSP), according to which if $A \to B$ is a theorem then the formulas A and B must share an atomic formula ('propositional variable') in common. This property has been informally motivated by appeal to talk of requiring an *overlap of meaning* to obtain between the antecedents and consequents of valid entailment claims, for instance in (Belnap, 1960) and (Anderson and Belnap, 1975). The most common examples given (for instance, that "2+2=5" does not entail "the moon is made of green cheese") seem to trade on shared meaning in terms of shared *topic*. According to this rough and ready construal, then, VSP requires that valid implications must not allow one to *completely* change topic.[1]

Logics of topic, which explicitly build in topic-theoretic machinery in the interpretation of certain *hyperintensional* operators, have recently come up for extended investigation, details of which may be found in (Berto, 2022). One way of constructing semantics for such systems employs, in addition to a ('Kripke') frame consisting of a set of worlds with an accessibility relation thereon, an explicitly specified *join semilattice* of topics. The join is understood to formally represent *topic fusion*, and the key idea is that one can always combine topics and obtain a new topic thereby. With the frame and the topic join-semilattice, one can then provide semantic treatments for a range of operators by distinguishing the *truth assessment* machinery (assigning formulas to sets of worlds which satisfy them) and the *topic assessment* machinery (assigning formulas to their topic), and the *interpretation* of a formula will then be a combination of these. The topic assignment is usually required to satisfy various properties, for instance requiring *topic transparency* for logical connectives. This requires that where t assigns formulas to topics and $\sqcup$ is topic fusion, we have identities like, for example:

$$t(A \wedge B) = t(A) \sqcup t(B)$$

In this framework, a number of operators with various properties have been fruitfully studied.

The aim of this talk is to investigate the topic-theoretic properties of relevant logics using some of the formal insights of contemporary logics of topic. In particular, the kinds of algebraic models used to prove that relevant logics satisfy VSP have precisely the kind of structure necessary to undergird such an investigation. It has been shown, in (Robles and Mendéz, 2012), that for a logic to have VSP it is sufficient that it be sound w.r.t. some matrix $\mathcal{A}$ where:

1. there exist distinct subalgebras $\mathcal{A}_1, \mathcal{A}_2$ of $\mathcal{A}$ and
2. if $\langle a, b\rangle \in \mathcal{A}_1 \times \mathcal{A}_2$ then $a \to b$ is undesignated.

With these conditions satisfied, we can find an interpretation of the language in $\mathcal{A}$ to falsify any $A \to B$ where A and B have no atomic formula in common: simply take an interpretation which interprets each atomic subformula of A in $\mathcal{A}_1$ and each atomic subformula of B in $\mathcal{A}_2$. It follows then that the interpretation of A will inhabit $\mathcal{A}_1$ and that of B will inhabit $\mathcal{A}_2$, since these are subalgebras, and so the formula $A \to B$ will be falsified, by condition 2.

[1]This should be contrasted with logics of analytic implication, which require valid implications not to *introduce* new topics in the consequent. Details can be found, for example, in (Ferguson, 2017).

In the case of De Morgan monoids (DMMs), the equivalent algebraic semantics for the relevant logic **R** introduced in (Dunn, 1966), we get a matrix definition from the unit element $\mathtt{t}$ (from the "monoid" part) and the lattice order $\leq$, fixing the designated values of $\mathcal{A} \in \mathsf{DMM}$ just to be $\{x \in \mathcal{A} \mid \mathtt{t} \leq x\}$. In any DMM we have that $\mathsf{t} \leq x \to y$ holds iff $x \leq y$, and so the latter of the two displayed conditions above can be simplified to "if $\langle a, b\rangle \in \mathcal{A}_1 \times \mathcal{A}_2$ then a is *incomparable* with b (i.e., $a \nleq b$ and $b \nleq a$)". So the kinds of DMMs needed to show that **R** has VSP are those where we have distinct subalgebras which are *incomparable* (to slightly abuse terminology). We have just such a structure in Belnap's M_0, as well as in Meyer's *crystal lattice*, investigated extensively in (Thistlewaite, McRobbie, and Meyer, 1988).[2] If we read VSP as topic-salient, then it seems quite natural to take DMMs to have a topic structure, delivered by such subalgebras. Furthermore, if we consider subalgebras as *generated* by collections of elements, then we obtain a *complete lattice ordering* of such subalgebras (or, more precisely, their carrier sets), as discussed in (Burris and Sankappanavar, 1980, Cor. 3.3). Furthermore, if we restrict our attention to subalgebras with *non-empty* carrier sets, then we get a complete join semilattice, with the join operator just being set union.

Seen this way, DMMs come along with a fairly natural topic structure delivered by this join semilattice of their subalgebras (with nonempty carrier sets). These subalgebras are then generated by the collections of propositions which are 'about' the topic in question (this motivates requiring nonemptiness, as a topic should have some propositions about it). In this talk, I shall begin to investigate DMMs, along with their subalgebras, from this topic-theoretic perspective and try to start building bridges between relevant logics and contemporary work in logics of topic.

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[2] A bit of caution is required here in how we define "subalgebra". DMMs are usually defined to incorporate an algebraic constant $\mathtt{t}$, as mentioned above, but this does not inhabit the carrier sets of the salient subalgebras of M_0, for instance, which are (using the notation of (Anderson and Belnap, 1975)) $\{+1, -1\}$ and $\{+2, -2\}$: in M_0, $\mathtt{t} = +0$. So we consider 'subalgebra' to apply to DMMs in the type *without* the $\mathtt{t}$ constant.

Modelling afthairetic modality

Pedro Yago
Scuola Normale Superiore di Pisa, Italy
e-mail: pedro.tyago@outlook.com

Giorgio Venturi
Università di Pisa, Italy
e-mail: gio.venturi@gmail.com

Arbitrary objects play an important role in mathematical practice. For example, they allow us to epistemically justify infinitary reasoning. However, they have interesting and peculiar characteristics. Two of them seem to be essential: the following of what has come to be known, following Kit Fine, the *Princilple of Generic Attribution* (the *PGA*), and the way in which they assume values. (Horsten, 2019) has advanced a particular view on arbitrary objects which describes how this assuming of values is in accordance with a sui generis modality, which he calls *afthairetic*. In this paper, we offer a logical framework, with a proof theory and semantics, for extending any first-order theory to a theory of its afthairetic modality.

Afthairetic modality may be best understood by considering a fair die. It makes sense to say a six *may* come up, or a two, or a three, but it does not make sense to ask what its *actual* value is. A fair die is an abstract object, it cannot be tossed for us to get an actual result. While it has six *possible* different values, neither of them are *actual*. In a similar fashion, arbitrary objects would assume values – never actually, but always possibly.

The description offered relies on how this specific relation of instantiation, the *assuming the value of*, changes from the actual world to possible worlds. Furthermore, the actual world seems to have a privileged position, for not only it is the only world in which no arbitrary objects assume values, but it also seems to access any possible world in which values are being assumed. In the same spirit, the proof system we develop focuses on describing the relation between arbitrary objects and their values, either actually or possibly, from the perspective of the actual world.

We translate the aforementioned descriptions into the language of first-order modal logic with identity, augmented with adequate predicates relating to arbitrariness and the assuming of values, in order to obtain the proper axioms, justifying each of them by recalling intuitive notions concerning arbitrary objects. Since the theory is to be an extension of a regular first-order theory, the language of the novel theory is to be an extension of that of the first-order theory, all all axioms and inference rules of it are inherited. Furthermore, we add a novel inference rule, which deals with two different notions of derivability: one for the theory of the new modality itself – which may be interpreted as the ”rules” of the actual, world –, and another for yet a different theory – the ”rules” of any other world –, such that neither one of the theories is an extension of the other. Furthermore, we argue the rule of *Necessitation* cannot hold, expressing the nonnormal character of the modality.

Intuitively, all of the characteristics of the modality suggest a semantic with designated worlds. By defining such semantics, we show conservation results from the original theory to the afthairetic modality theory, both in the proof-theoretic and the semantic front. We then prove the soundness and completeness of the minimal system. Having those results which show the adequacy of our system, what we find is an interesting accessibility relation: one in which there is always at least one world accessible from the actual world, no world accesses the actual world, and in which the accessiblity relation can never be reflexive.

We move on to provide possible extensions of the minimal system which incorporate Horsten's criterion of identity for arbitrary objects, a comprehension scheme concerning their abundance, and a principle of relation inheritance (the *PRI*) – which is merely suggested by Horsten, but which offers interesting insights into how a more general account may be offered –, borrowed from the work in (Linnebo, 2018). We show the extensions are sound and complete with respect to the appropriate classes of models of our semantics. We then elucidate some of the philosophical commitments our framework poses for Horsten's view, and compare them to Kit Fine's view in (Fine and Tennant, 1983) and (Fine, 1985) – more specifically, the similarities and differences between the accounts of identity, and of the PGA, as opposed to the PRI.

We conclude with some remarks on what that means for the concept of arbitrary object, and plans for future work.

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