What Are Logical Forms (Good for)?

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ABSTRACTS

Catch me if you can
– on the ineffability of logical form with Wittgenstein and others

Pavel Arazim

The legendary pianist Glenn Gould has praised Symphony number 5 by Jean Sibelius as one of the most radical form-as-process experiments. This means that he considered the form of this composition as not preceding its content but rather happening together with the content. Saying this about the form of symphony, in fact of any musical composition, is just as paradoxical as would saying it of a logical form be.

Typically, one would guess and could very well support such an intuition by Russell´s understanding of logical analysis, form is supposed to precede the content. It should be a container ready to be filled with various contents. Gould seems to reverse this order and to praise this reversal as he perceives it in Sibelius. He regards this reversal as the source of the particular force and authenticity of this great symphony.

Can we transpose this view from the realm of music into that of logic? And what would it mean for the possibility of studying logical form, as it is attempted by formal logic? Wittgenstein doubts already in the Tractatus the possibility of expressing logical form, as it would mean going outside our epistemic framework.

This view is only strengthened by his focus on language games. Games are so variegated that it is impossible to come up with a satisfactory characterization of them. His important colleague and inspiration during his transition from the Tractatus to the late philosophy, Moritz Schlick, also characterizes games as activities which bear their purposes in themselves. We do not play to achieve something in a similar way in which we work to get our salary. Games are played for themselves, they do not need a justification. And Wittgenstein sees our language as a game or motley of games.

As such, it does not have an essence. Or the essence is continuously created by our decisions and agreements which do not need justifications. In this way the meanings of our statements do not have a stable underlying logical form which only waits to be unearthed. To the contrary, they get their form only in the process of being used. The attempts to find the logical form of our statements is, therefore doomed to failure. Logical nihilism, the thesis that every logical system is wrong, is thus closer to Wittgenstein´s wisdom than logical pluralism, claiming that many systems are correct and help us discover logical forms of our statements in various areas of discourse and in different contexts.

Logical Forms: Validity and Unity

Georg Brun

This talk discusses logical forms of sentences as they are represented by formalizations in first-order logic. I first show why the standard practice of formalizing should make us cautious about speaking of *the* logical form of a sentence or inference. The reason is that the standard practice admits that a sentence has a plurality of adequate formalizations, which can be related to each other in various ways: by being notational variations, by equivalence or by being more or less specific. On the other hand, the standard practice of showing sentences or inferences to be valid also presupposes that formalizations of the same sentence constitute a certain unity. As I argue, this unity is best understood as constituted by a hierarchical structure of formalizations related by substitutions. Postulating such a hierarchy has not only consequences for the question of which formalizations count as adequate, but also underwrites the well-known asymmetry between verdicts of formal validity and verdicts of formal invalidity. This picture of the standard practice of formalizing is then compared with alternative views which result from stronger assumptions about the unity of formalizations, specifically requirements that ensure that all formalizations must have the same validity status.

Analytic vs. Logical Validity

Marie Duží

My starting point is this; an *analytically true* sentence is true solely in virtue of its meaning. Yet, this simple specification gives rise to questions. First, what does it mean 'solely in virtue of its meaning'? Second, if a sentence is analytically true, is it provable in a sufficiently rich proof calculus? Third, what is the difference between analytically and logically true sentences?

To answer these questions, I apply the system of Transparent Intensional Logic (TIL) with its *procedural* semantics. The meaning of an analytically true empirical sentence is the procedure that produces the proposition TRUE that takes the value true in all possible worlds and times, and the meaning of an analytically true mathematical sentence is the procedure that produces the truth value T regardless of worlds and times. Yet, these meaning procedures do not have to produce the truth in an efficient, computable way. If so, we say that the sentence is analytically but not logically true, and it is in general not provable. The definition of a *logically true* sentence is inspired by the traditional Bolzano's doctrine. To this end, I first define the *literal analysis* of a sentence, which is afterwards transformed into its *logical form*; this is obtained by substituting free variables (not occurring so far) for all the constituents that refer to non-logical objects. The sentence is *logically true* if its logical form produces the proposition TRUE (or truth value T in the case of mathematics) for all the valuations of its free variables. Since TIL is a hyperintensional, typed lambda calculus, it cannot be complete in Gödel's sense. Yet, it is possible to define its intensional subsystem complete in Henkin's sense, i.e. with respect to *general models.* These models consist of algorithmically computable functions.

Singularism, Pluralism, and Definitional Equivalence

Salvatore Florio, Stewart Shapiro, and Eric Snyder

Atomistic classical mereology and plural logic provide two alternative frameworks for the analysis of plurals in natural language. It is a matter of dispute which framework is preferable. From the formal point of view, however, there is a strong sense in which the two frameworks are equivalent. So they have the same coverage as each other: there is a range of data about plurals that they both capture correctly. We argue that the tie is broken when we consider a wider range of linguistic phenomena, such as mass nouns and group nouns. Mereology is more flexible than plural logic and is thus more easily adapted to account for these richer fragments of natural language.

The Vagueness of Logical Forms

Zack Garett

If there is a traditional view on logical forms it is that the logical form of a sentence can be discovered by keeping fixed the logical constants in the sentence and abstracting away all of the other content. This idea dates back at least to Bernard Bolzano. In his *Wissenschaftslehre* Bolzano defines “logically analytic propositions” as propositions that are true regardless of how the non-logical ideas in the proposition are replaced. A logical truth—or a logically analytic proposition—is true by virtue of its logical structure, and its logical structure is given by the logical constants. There is, however, a problem with the traditional view. The boundaries of the concept “logical constant” are not precise. Just like the word “bald,” “logical constant” is vague. In this paper, I first provide a sorites argument to demonstrate the vagueness of the category of logical constants. One response to the traditional view, the response of the debunkers, claims that there is no sense in talking about *the* logical form of a sentence. According to the debunkers, the fact that there is no way to draw a principled line between logical and non-logical words opens up the line to be drawn wherever it is practical to do so. Debunkers claim that the subject matter of logic is not inferences that are correct because of logical form, but instead, inferences that are correct *simpliciter*. By drawing the line for “logical constant” in different places, we can capture different sets of correct inferences. This approach to the vagueness of the concept “logical constant” resembles theories of vagueness that relativize truth to precisifications. In the second part of this paper, I argue that the debunkers’ position faces a problematic dilemma—one faced also by the analogous theories of vagueness. On one horn, they must detach the word ‘logic’ from both its ordinary and technical meanings. On the other horn, they must limit what can count as a logical constant, and thereby fail to avoid the vagueness of the concept.

Bolzano notes the imprecise nature of the concept “logical.” Writing about the distinction between analytic truths and logical truths, Bolzano states in §148 of *Wissenschaftslehre*, “To be sure, this distinction has its ambiguity, because the domain of concepts belonging to logic is not so sharply demarcated that no dispute could ever arise over it.” Bolzano refers to the line between merely analytic propositions and logically analytic ones as ambiguous. Not only are there borderline cases of logical constants, but the concept “logical constant” obeys a tolerance principle.

Jc Beall defines full tolerance as follows: If x and y stand in some tolerance relation with respect to (vague) F, then they both satisfy F or neither does (Beall 2009, 187). Tolerance is demonstrated clearly in the construction of a sorites series. Two adjacent individuals will either both instantiate the vague property or both fail to instantiate it. William Lycan (1989) provides the beginnings of a sorites series for logicalness. His series, however, does not clearly demonstrate a tolerance principle. Uncomfortable lines can be drawn in his series, but the lines that can be drawn do not seem as problematic as the lines that an epistemicist draws in sorites series that demonstrate tolerance.

A sorites series can be created, and doing so helps demonstrate that the concept "logical constant" is not just ambiguous; it is vague. The dimension on which the concept "logical constant" is vague is topic-neutrality. Consider the word "and" as it is defined in the classical truth-tables. There is a sense in which the word 'and' is topic-neutral. It outputs true or false depending only on the truth-values of its conjuncts—the particular contents of the conjuncts are irrelevant. This truth-functional understanding of 'and' is not without its controversy. Dorothy Edgington (1992) argues for a degree-theoretic but not degree-functional semantics for 'and.' She argues that the truth of a conjunction is not just dependent on the degrees of truth of its conjuncts, but also on other relations that hold between the conjuncts. Edgington's conjunction is not topic-neutral to the same degree as the truth-functional 'and.' Though her conjunction is not fully topic-neutral. Taking the position that it is not a logical constant would be incredibly extreme.

If Edgington's conjunction is a logical constant, then we have the beginnings of a sorites series. I will provide this example in the context of a continuum-valued logic. This is not to say that such a logic is the correct logic, but sticking with continuum-valued logics provides the clearest version of the example. Let $\vee_0$ be a binary operator on sentences, and let $c(\phi, \psi)$ be the degree to which ϕ's being true causes ψ's being true. If ϕ is a cause of ψ, then $c(\phi, \psi) = 1$, otherwise $c(\phi, \psi) = 0$. $\vee_0$ is defined by the following truth conditions:

$$\mathcal{v}(\phi \vee_0 \psi) = \max(\phi, \psi)(1) \dot{-} c(\phi, \psi)(0)$$

$\vee_0$ is no different than a normal continuum-valued disjunction. Its value is equal to the maximum degree of truth of its disjuncts, and so it is topic-neutral in the same way as a degree-functional disjunction. It is clearly a logical operator.

I will now define a template for other sentential operators similar to, but not the same as, $\vee_0$.

$$\mathcal{v}(\phi \vee_n \psi) = \max(\phi, \psi)(1 \dot{-} n/1000) \dot{-} c(\phi, \psi)(n/1000)$$

For each of the versions of $\vee_n$ we have a different weighting for the influence of the disjuncts and for the influence of the logical relationship between the disjuncts. For example, at $n = 400$, if ϕ is a cause of ψ and ϕ is true to degree 1, then $\phi \vee_{400} \psi$ will be true to degree .2. Because the influence of the causal relationship grows and the influence of the truth-values of the disjuncts diminishes as we go from 0 to 1000, we eventually end up with a sentential operator that is only about a causal relationship. Unless we want to treat causal relationships as logical relationships, $\vee_{1000}$ is not a logical operator. $\vee_1$, however, should still count as a logical operator. After all, we may be concerned, as Edgington is, with the influences our disjuncts may bear to one another. We want to say that at least one of the disjuncts is true, but at the same time downgrade the truth of the disjunction when ϕ is responsible for ψ's being true. The amount to which we want to downgrade the truth of the disjunction may vary, even within a mostly logical context.

P1. $\vee_0$ is a logical operator.

P2. If $\vee_n$ is a logical operator, then so is $\vee_{n+1}$.

C. $\vee_{1000}$ is a logical operator.

The sorites argument here demonstrates the tolerance principle. When two of the V_n connectives are close enough together in the series, then both count as logical operators. Of course, sorites arguments are not usually accepted. Either one of the premises is rejected or the argument is shown to be invalid in some non-classical logic. There are two approaches to vagueness I want to discuss here because versions of them have become common responses to the vagueness of logical constants.

Nihilism as an approach to vagueness takes the sorites paradox as a reason to reject vague concepts. Peter Unger (1980), for example, treats vagueness as a reason for rejecting the existence of ordinary objects. A similar approach is applied to the concept 'logical constant.' A position like this holds that there are no privileged logical words. Under a nihilist theory, there is little sense in talking about logical truths as distinct from other kinds of analytic truths.

Another approach to vagueness that has garnered attention recently relativizes truth to precisifications. Examples of this strategy can be seen in Diana Raffman's (2013) multi-range theory and Nicholas J.J. Smith's (2008) degree theory. Unlike supervaluationists Raffman and Smith do not read the truth-value of a sentence from the ratio of true to false precisifications. Instead, they claim that sentences are merely true or false relative to a precisification. This approach appears in the literature on logical constants. There is little sense in talking about *the* logical form of a sentence. Instead, we can talk about its logical form given certain cutoffs for the classification "logical constant."

Both of these approaches to vagueness have in common a push towards treating distinctions as a matter of practical concern. There may not be any objective line to be drawn between when a vague predicate applies and when it doesn't, and so the proper use of those vague concepts is just a matter of what use we can make of them. In the domain of logical constants and logical forms, we can see this kind of approach in, for example, Lepore and Ludwig (2001). They treat the logical form of a sentence as its semantic structure and throw out the distinction between logical and non-logical words. The result is an account of analyticity. They claim that one could designate a special set of words to get a subset of the analytic truths, but doing so would only serve a practical purpose.

The problem with these approaches to vagueness is that the only way they can avoid vagueness is by throwing away the ordinary meanings of vague words. If the line between the logical and the non-logical can be drawn for practical purposes, then it can be determinately outside of the bounds of the ordinary meaning of "logical." One who wishes to discuss logical forms, then, can make the claim that the nihilist or the relativist about vagueness has just stopped talking about the same concept. If like, Smith, we put a limit on which precisifications are acceptable—if we limit precisifications to ones that are in line with ordinary meaning—then we haven't escaped the vagueness of the concept.

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Logical Form, Causal Efficacy, and Abstract Objects

Deke Gould

On the standard view, abstract objects are causally inefficacious (Lewis 1986, Wetzel). I will argue that this standard view is incorrect. I argue that logical form should be taken to be causally efficiacious (Callard 2007, Park 2018). I begin by sketching an argument for a platonist ontology of logic (Berry 1968, Balaguer 1998, Katz 1980). Then, I consider recent disputes about the causal efficacy of abstract objects in other domains (Dodd 2007, Juvshik 2018). There are some areas where such causal exclusion arguments appear to be unsuccessful, especially with regard to the normativity of logic and the role of logical form in thought (Besson 2019, Milne 2009, Russell 2017). I argue that viewing logical form as playing a contributory causal role in thought can best account for logic's normativity (Cresswell 2010, Popper 1978). One significant challenge for the idea that abstract objects are causally efficacious concerns the assessment of counterfactuals involving those objects (Lewis 1973). A natural view maintains that all such counterpossibles are vacuous (Williamson 2007). After all, abstract objects of the relevant sort are necessarily existing entities. However, recent work on modal frameworks incorporating impossible worlds offers methods for showing that not all such counterpossible conditionals are vacuously true (Berto et al. 2018, Nolan 1997). By applying such a framework to the ontology of logic, the platonist has the opportunity to provide a non-vacuous account of the truth of counterpossibles involving abstract objects. I explore such a platonist account of the causal efficacy of logical form, and I conclude by considering how such a view might help address both the well-known Benacerraf problem and the reliability challenge in the epistemology of logic (Benacceraf 1973, Schechter 2010).

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On the Formalization of Conditionals

Andrea Iacona

This paper deals with some thorny issues that concern the formalization of conditionals. In particular, it discusses three kinds of cases in which there is no clear answer to the question of how a sentence that contains `if' or similar expressions is to be represented in a formal language that includes a symbol for the conditional. As it will be suggested, the examples discussed show in different ways that an adequate formalization of a sentence must take into account the content expressed by the sentence. This is exactly what one should expect on the view that logical form is determined by truth conditions.

Ambiguous interpretations, Logical Form and Identity

Daniel Molto

In this paper, I argue in favour of the possibility of ambiguous interpretations. By 'ambiguous interpretations' I do not merely mean interpretations according to which terms of the object language are ambiguous, but rather interpretations that are themselves ambiguous, but nevertheless genuine interpretations. I consider three options here: (1) that any attempt to state an interpretation which involves metalinguistic lexical ambiguity is always a failed attempt at expressing an interpretation; (2) that some attempts at interpretation which involve metalinguistic lexical ambiguity express one or more genuine interpretations, but that those interpretations themselves are always determinate and unambiguous; and (3) that some attempts at interpretation which involve metalinguistic lexical ambiguity express genuine interpretations, and that these interpretations are themselves ambiguous (in a sense to be made clear). I argue for (3).

The position I will be defending might in turn lead one to a further view, namely that if the logical form of a sentence is one which, by stipulation, eliminates all lexical ambiguity, then accessing the logical form of a sentence is not a necessary preliminary for interpreting a sentence. Indeed, it could well be that the logical form of a sentence is inaccessible in principle, and yet the sentence is still interpretable. Logical forms, on this view, would not play the central role in interpretation that it is often thought they do. Alternatively, one might instead draw the conclusion from (3) that sentences in their logical form may still be lexically ambiguous. This runs counter to common view that the one of the characteristics of logical form is that it eliminates ambiguity. In either case, my defence of (3) tells us something important about logical form.

I hold that interpretations can be ambiguous in their specifications of the semantic values of predicates, singular terms and variables. The result of allowing such ambiguity can tell us a lot about topics such as the indeterminacy of translation (Quine 1960) and meaning (Davidson 1984), conceptual engineering (Floridi 2011), and the nature of identity relations (Geach 1962). I finish by drawing a connection between my thesis and theories of relative identity in particular.

Logical forms of what?

Jaroslav Peregrin & Vladimír Svoboda

We suggest that the questions *What are logical forms?* and *What are logical forms good for?* are so closely related to questions *What is logic?* and *What is logic good for?* that every attempt at presenting a satisfactory answer to the former questions should be preceded by at least a short outlining of answers to the latter ones. As an introductory step towards providing the needed answers we begin with disambiguating of the term "logic". We distinguish three meanings that the term bears in the contemporary literature on (philosophy of) logic: the term is used to denote a specific discipline, to refer to a phenomenon or an aspect of reality that the discipline studies. Finally, the term refers to individual theories developed within the discipline – logics (like for example classical first-order predicate logic).

We then briefly survey different possible accounts of the subject of the study of logic. Subsequently, we outline the picture to which we adhere - we maintain that logic, as we see it, is neither concerned with ideal or mental entities like the tokens of an (alleged) language of thought, nor it is to be seen as some general structure of the natural world. We claim that roots of logic should be sought in natural languages as they evolved during the evolution of the human race - in specific structural features that characterize any developed natural language.

However, there are different kinds of languages: besides natural languages, there are artificial languages produced by us. We distinguish different sorts of such languages as they appear in logic and related areas and argue that it is the sentences and arguments articulated in the natural ones that are to be seen as *having* logical forms, while sentences and compounds of sentences of the artificial ones – especially the formal languages of logical systems - serve as (the embodiments of) the logical forms. We can imagine that the formal languages serve as models of the natural ones and assigning a logical form to the sentence is in fact pinning up the sentence to a particular point in the model. This helps us see the "inferential neighborhood" of the sentence, and thus help us to decide on (logical) correctness/validity of arguments and systems of arguments (in different analytic projects) and disambiguate sentences and texts that are obscure in natural languages.

Our overall picture of modern logic suggests that artificial languages that are created and studied by logicians are inevitably rooted in natural languages – they primarily serve as models that are meant to expose and standardize specific features of natural languages, in particular those that play a crucial role in argumentation. The primary objects of logical analysis are texts in natural languages (such as arguments and systems of arguments). The analysis should be always seen as a purpose oriented enterprise rather than as discovering facts.

Canonical and noncanonical forms of proofs

Ivo Pezlar

Logical forms can help us distinguish valid inferences from the invalid ones. But not all valid inferences are always viewed as equal. Some are preferred to others, e.g., in the inferentialist tradition, an inference of a conjunction of the form $\frac{A \quad B}{A \wedge B}$ is preferred to an inference of the form $\frac{C \to (A \wedge B) \quad C}{A \wedge B}$. Both inferences are valid, both result in the same conclusion, but the former is preferred because the logical form of the conclusion is important for the validity of the whole inference form. This is not the case for the latter inference, where the proposition $A \wedge B$ could have been replaced by a proposition with any other logical form and the resulting inference form would still be valid (although it would no longer be an inference of a conjunction).

This distinction between what is typically called canonical and noncanonical forms of inferences or proofs (more generally, between direct/primitive and indirect/defined justifications) plays a crucial role in many proof-theoretic theories of meaning (e.g., Dummett (1991), Martin-Löf (1996)). It is a powerful explanation tool that allows us to conceptualize, e.g., the distinction between meaning and denotation (in Fregean sense), between a computation and its value (within the propositions as types paradigm), or to escape the paradox of implication meaning explanation (in constructive semantics). Furthermore, the reduction of noncanonical proofs into canonical ones can be seen as a formal counterpart of Dummett's fundamental assumption regarding proof-theoretic justification of logical laws.

In this talk, we will examine this distinction that makes some valid forms of inferences more desirable than others. We will be interested not only in determining what constitutes canonical forms of proofs (which can be paraphrased as: what are introduction rules? as introduction rules often play a central part in their specification) but especially in what happens when the connection between noncanonical and canonical proofs is severed. This situation is rarely explicitly considered but, among other things, it can give us tools to reason about paradoxes in proof-theoretic setting (see, e.g., Tennant (1982), Tranchini (2019)). It remains, however, an open issue how to understand the notion of a noncanonical proof that is irreducible to a canonical one as the meaning of the former is typically regarded as constituted by its reducibility to the latter. To phrase it differently, what sort of knowledge, if any, can an indirect justification that cannot be made direct convey?

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Logic as Applied Mathematics—with Particular Application to the Notion of Logical Form

Graham Priest

The word ‘logic’ has many senses. Here we will understand it as meaning an account of what follows from what and why. With contemporary methodology, logic in this sense—though it may not always have been thought of in this way—is a branch of applied mathematics. This has various implications for how one understands a number of issues concerning validity. In this talk I will explain this perspective of logic, and explore some of its consequences, notably with respect to logical form.

Logical Forms, Substitutions, and Types of Information

Vít Punčochář

The common practice of modern logic is to employ an artificial language whose expressions do not represent concrete sentences but rather their forms, and accordingly they are called formulas. Logical semantics provides a space of possible interpretations, i.e. possible ways in which formulas can be filled in with content. Some formulas are true throughout all interpretations and thus, in a clear sense, independently of any particular content. Such formulas represent logically valid forms, and concrete fully-fledged sentences that we subsume under these forms can be called logically true (at least from the perspective of the given logical theory). This I believe is the received view of logic.

There is one principle that is interpreted as a technical counterpart of the claim that logic is a matter of form, and that is the principle of uniform substitution. Given a standard language of propositional logic, a substitution is a function that assigns a formula to each atomic formula. These assignments can be "homomorphically" extended to the whole propositional language and thus substitutions can be viewed as functions assigning formulas to all formulas. A set X of formulas is closed under uniform substitution if for every formula A, and every substitution s, if A is in X then also $s(A)$ is in X.

It seems that the collection of all logically valid formulas should be closed under uniform substitution. If $s(A)$ is not logically valid then $s(A)$ can be filled in with a content that makes it false. But then it seems that also A can be filled in with a content which makes it false and thus A itself should not be regarded as logically valid. And indeed the usual technical definitions of "logic" often incorporate the principle of uniform substitution among the most basic requirements. For example, a set of formulas is called a superintuitionistic logic if it contains all intuitionistically valid formulas, and is closed under modus ponens and uniform substitution. The principle of uniform substitution is also a cornerstone of modern algebraic logic [3].

The most common logical systems are closed under uniform substitution. But there are significant exceptions: for example, Carnap's modal logic C [1], Veltman's data logic [6], public announcement logic [5], and inquisitive logic [2].

In my talk I will argue that these examples show clearly that the principle of uniform substitution does not capture adequately the requirement that logic is a matter of form and that logical truths are formal truths. I will argue that particular logical expressions can produce propositions of different kinds and the resulting diversity of informational types can lead to a justified failure of uniform substitution without undermining the view that logic is purely a matter of form. A similar position was advocated by Schurz in [4].

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On the Logical Form of Some Conditionals

Hans Rott

Recently, a number of authors have expressed dissatisfaction with the dominant analyses of natural-language conditionals as "suppositional conditionals". The latter are true or accepted if the consequent is true/accepted on the supposition of the antecedent. But this can happen although the antecedent is completely irrelevant (or even somewhat adverse) to the consequent. In natural-language conditionals, however, the antecedent is typically meant to *support* or *be evidence* for the consequent. The logical form of conditionals will thus be more complex than the suppositional theory would have it. Recently a few suggestions as to how this logical form might look like have been made. In this talk, I compare Vincenzo Crupi and Andrea Iacona's account of "evidential conditionals" with my own account of "difference-making conditionals" and "dependence conditionals". While the characteristic feature of the first is the validity of *contraposition*, those of the second and third are the validity of *conjunctive rationality* and *conditional perfection*, respectively. Time permitting, I will also touch on concessive conditionals and the connective 'because' (the "factual conditional", according to Goodman).

Formality through the lens of pragmatic genealogy

Gil Sagi

This talk will concern the concept of logical consequence, and the concept of logical form in the service of the study of logical consequence. The methodology used is inspired by that of *pragmatic genealogy* (Queloz 2021). First, this entails a functionalist approach: we ask not "What is logical consequence?", but "What does logical consequence do for us?" Secondly, we give a dynamic model that is partly fictional and partly historical to uncover the roots of the concept: the basic motivations that may lead to its current practice. Finally, we use the analysis to separate core and dispensable aspects of current conceptual practice.

Our methodology thus includes reverse-engineering from the conceptual practice to its function, and re-engineering from the function to a new conceptual practice. In both cases, there is more than one way to go. We offer a narrative that uncovers in our conceptual practice of logical consequence the motivation for having a safe and practical mode of reasoning, and that further recommends new formal frameworks for the practice.

Our focus will be on the formality of logical consequence, and with it on logical form. The conceptual practice associated with logical form, from the present perspective, branches out of the main one, having to do with logical consequence. The concept of logical form is conventionally entwined with that of logical term. While it will be shown that formality is a well-motivated condition on logical consequence, its manifestation through the distinction between logical and nonlogical terms will be shown to be merely contingent. We offer a new semantic framework, of *semantic constraints*, where a strict division of the vocabulary between logical and nonlogical does not play a primary role. A re-evaluation of the notion of logical form is then called for.

Natural Logical Truth versus Satisfiability in All Models

Gabriel Sandu

In Hintikka and Sandu (In Dale Jacquette (ed.), *Philosophy of Logic*. North Holland. pp. 13--39 (2006)) we argued against an inferential account of logic and defended a descriptive account according to which our basic logic is adequate only to the extend it is able to describe specific notions (e.g. infinity, etc). We argued in that paper that that view of logic is best spelled out withing the model-theoretic paradigm (descriptive complexity). In the present paper I will go beyond the model-theoretic account and consider a “natural” conception of logic inspired by Russell. I will describe its basic features (based on J. Almog, V. Halava and G. Sandu, “Logic and the ultimate completeness ideal. In memory of Hilary Putnam”, forthcoming) and consider some of its consequences for inferentialism and the question of the adequacy of a formalization.

Logical constants & logical form

Sebastian G.W. Speitel

An influential tradition in the philosophy of logic holds that the *logical form* of a sentence or argument is determined by its logical constants. What is a logical constant, however, and thus which forms are logical, is a difficult question to answer. In a previous paper [1], we argued that for an expression to be a logical constant denotational and inferential aspects of its meaning must align in a particularly favourable way. We proposed a *combined criterion of logicality* based on the idea that logical notions must be formal and categorical (uniquely determinable by inference). The resulting delineation of logical forms exceeds the boundaries of standard first-order logic and renders several novel forms of sentences logical.

In this talk, I want to reflect further on the limits of logic and logical form set by this criterion and discuss some of its consequences. In light of the fact that the notion "there are infinitely many" now qualifies as logical, for example, the structure of the natural numbers becomes categorically characterizable. Does this imply that arithmetical forms have been reduced to logical forms? Furthermore, what about forms that qualify as logical according to the criterion relative to holding other notions fixed? Are these forms logical in their own right, or does their status depend on the status of the notions on which they rely?

Different answers to these questions are possible. In this talk I wish to discuss their merits and some of their consequences for the notion of logical form as traditionally conceived.

[1] D. Bonnay and S.G.W. Speitel, "The Ways of Logicality: Invariance and Categoricity", in: *The Semantic Conception of Logic: Essays on Consequence, Invariance, and Meaning*, G. Sagi & J. Woods (eds.), Cambridge University Press 2021, pp. 55-80.

About Logical Hylomorphism

Karel Šebela

Since its founding at Aristotle, logic has shown a striking focus on the form of argumentation. This distinction between form and content still plays a key role in logic, not coincidentally called hylemorphism in logic. In modern logic, the emphasis on form has prevailed so much that formal logic has almost become synonymous with logic, and the times when we can find some kind of a material logic in logical textbooks seem to be almost forgotten. In this talk, I will focus on the relationship of this proclaimed formality of logic to the hylemorphic framework. Formality of logic is frequently taken as a key feature of logic, so I follow the conceptions of logic by Plato, Aristotle, peripatetic and neoplatonistic commentators of Aristotle and early scholastics, until Abaelard. Surprisingly enough, Aristotle is not the father of logical hylomorphism, hylomorphic terminology appears as far as in the work of neoplatonic commentators of Aristotle. But the key role here is the Abelard’s work. Abelard's distinction between perfect and imperfect inferences, and especially his reasoning for this distinction, became canonical. Although the terminology has changed, in logical tradition the fundamental difference between inferences, the validity of which, in a more modern vocabulary, is given a priori and by the others has been preserved since his time. The former assumed that they were valid on the basis of form, and the view spread that these were the very subject of logic, that logic is actually formal logic. Dependence on the world is a sign of non-logicality, materiality. It is not necessary for the logic thus cultivated to be Platonist (it may be a version of nominalism), in any case contact with the "nature of things" is not necessary here. So, modern logic is purely formal and its way of existence is the thinking of pure form. Originally, the Aristotelian dichotomy of matter and form paradoxically became the path to its platonisation.

An Expressivist Strategy to Understand Logical Forms

Giacomo Turbanti

In this paper I intend to discuss a way in which logical forms can be understood from the point of view of logical expressivism. I will defend two claims:

A. The core expressivist thesis is neutral with respect to different and possibly incompatible theories of inference.

B. Logical expressivism offers a natural framework for understanding logical forms and their function.

First, I will consider three fundamental questions that originally emerged in the modern debate about logical forms and that any account of logical forms should address. Then, I will define an expressivist strategy to answer these questions. Finally, I will apply this strategy to three paradigmatic cases.

For the purposes of the present paper I will use a relatively loose notion of logical form to refer not only to patterns of logical constants and variables expressing inferential properties, but also to the meaning that logical vocabularies acquire by occurring in such patterns.

Three questions about logical forms

From a historical point of view, the basic logical forms of the standard languages of modern logic were defined in the period roughly between Frege's *Begriffsschrift* and Carnap's *Logical Syntax of Language*. As is well known, however, the process that led to such definition was far from linear or uniform. I suggest distinguishing between at least three intertwined but theoretically distinct questions that were involved in those original reflections.

1. A question about the *nature* of logical forms: What do logical forms consist in?

2. A question about the *justification* of logical forms: Why are only certain logical forms correct?

3. A question about the *analysis* of logical forms: What are the correct logical forms?

These questions were by no means specific to the debate in logic at the turn of the twentieth century. To the contrary, they largely preceded it and are still central to philosophical logic and metaphysics today. That they are in effect distinguishable is shown by the fact that they can be and have been answered independently in different ways.

The expressivist point of view

Logical expressivism was originally defined in the context of Brandom's normative inferentialism, with relation to the notion of pragmatic metavocabularies (Brandom, 2008). However, I apply it here in a more general form. In this wide or mild sense, logical expressivism can be defined as the thesis that logical vocabulary is characterized by the expressive role of making explicit inferential relations between the contents expressed by non-logical vocabulary. As a general thesis about the expressive role of logical vocabulary, logical expressivism is in principle uncommitted with respect to any of the three kinds of questions about logical forms distinguished above. Instead, it provides a strategy to clarify the different views that emerge from the different answers that can be given to those questions.

The first step of this strategy consists in the elaboration of a theory of inference. Such a theory establishes what logic is about and, therefore, the nature of logical forms. The latter are then conceived as expressive resources to make explicit the structural properties that make certain patterns of inference valid according to the theory. Consequently, the only justified logical forms are those that make those properties explicit and the correct analysis of logical forms must be have such an expressive task in view.

This strategy is general enough to work with different theories of inference. Here are a couple of examples.

In the representationalist view, for instance, logic is about the structure of reality. An inference is valid because any way for the premises to be the case is also a way for the conclusions to be the case. Logical forms make explicit that certain states of affairs have a certain structure in virtue of which they can, must or must not be the case together. Therefore, the logical forms that make explicit only the structural properties of reality are correct.

In the inferentialist view, instead, logic is about the structure of inferential practices. An inference is valid because it is treated as such according to the rules of the practice. Logical forms make such rules explicit and, therefore, the correct logical forms are those that make explicit only the rules of the inferential practice.

Applications of the expressivist approach

In what follows, I will discuss the application of the expressivist strategy presented above to the interpretation of three cases in which new logical forms were developed whose use delivered particularly significant proof-theoretical results.

I The first case is the motivating example in the original presentation of logical expressivism (Brandom, 1994, 107–116). It is the case of propositional connectives. In the expressivist approach, these are seen as expressive resources that allow to say explicitly what is only done implicitly by treating the conditions for the assertion of certain contents as related with one another. These conditions and relations may vary depending on the theory of inference that it adopted. The following, for instance, is a fairly common set:

- The assertion of a conditional like "$p \supset q$" makes explicit that q is treated as following from p: it says that if p is asserted then q cannot be denied.
- The assertion of a negation like "$\neg p$" makes explicit that that content is treated as minimally incompatible with p: it says that if $\neg p$ is asserted, then p cannot be asserted and *viceversa*.
- The assertion of a conjunction like "$p \wedge q$" makes explicit that both p and q are treated as forming a single content: it says that one cannot deny either p or q.
- The assertion of a disjunction like "$p \vee q$" makes explicit that either p or q are treated as contributing to a single content: it says that one cannot deny both p and q.

As Frege notices in the preface of his *Begriffsschrift*, the possibility to make relations between contents explicit as judgments is an essential condition for the study of inferences and their validity (Frege, 1879, 5).

II The second case is the logical form of Gentzen's sequent calculus. Sequents are an expressive device that allows to say what is implicitly done by treating some contents as following from others. The assertion of a sequent like "$\Gamma \Rightarrow \Delta$" makes explicit that the sequences of formulas Γ and Δ are treated respectively as the antecedents and the consequents of an inference. In other words, the logical form of sequent calculus allows to express as a judgment that a certain series of sequences of formulas is treated as a proof.

Sequent calculus makes it possible to do a proof theory where proofs themselves are the objects of study in a specific, explicit sense (Kreisel, 1971). This can easily be seen by comparing cut elimination in sequent calculus with normalization in natural deduction (Prawitz, 1965, 1971). Both results follow by establishing equivalences between derivations in sequent calculus and natural deduction respectively, but they actually say different things in completely different ways:

- The rules of natural deduction establish that certain formulas follow from other formulas. Normalization proceeds by showing that a given procedure for deriving a certain conclusion from certain premises by applying certain rules can be transformed into another procedure for deriving the same conclusion from the same premises by applying different rules: it shows that in doing the former one is also doing the latter.

- The rules of sequent calculus already establish that certain proofs can be transformed into other proofs. Cut elimination proceeds by showing that these transformations can always be performed without losing content by means of the cut rule: it shows that in saying that the proofs to be transformed without the cut rule are good then one is also saying that the transformed proofs are good.

It is well known that Gentzen developed sequent calculus to prove the *Hauptsatz*, by overcoming what he thought to be limitations in the logical form of his natural calculus for classical logic (Gentzen, 1934/35, 289). Of course, Prawitz's results of normalization showed that natural deduction has no intrinsic inconvenience in that respect. There is nonetheless an expressive gap between the two systems: while natural deduction only allows to implicitly show how to do proofs by applying rules, sequent calculus allows to explicitly talk about proofs as formulas.

III The third case I consider is Belnap's display logic. In such a framework, sequents $X \Rightarrow Y$ are composed of structures X, Y that are built out of formulas by using structural connectives. These connectives make explicit the properties of the structure of inferences that determine the rules that govern logical operators. Gentzen's sequent calculus already distinguishes between logical rules applied to complex formulas and structural rules applied to sequences of formulas. By operating on sequences one implicitly treats the way in which formulas are taken together as having inferential significance. In fact, substructural logics show how the meaning of logical operators varies depending precisely on how sequences are handled. There are three main aspects of the use of sequences in sequent calculus that have inferential significance:

- First, sequences can occur as antecedents or as consequents in a sequent; formulas occurring in sequences in the antecedents are treated as having a positive polarity, while formulas occurring in sequences in the consequents are treated as having a negative polarity.

- Second, sequences can group formulas together by means of the ","; sequences of formulas grouped together indicate a single content and are treated as conjunctions or disjunctions depending on their polarity.

- Third, sequences can be empty; empty sequences indicate null contents and again are treated as truth or falsity depending on their polarity.

Belnap (1982) introduced three structural connectives to make these significances explicit:

- A binary connective "∘" to fuse formulas together in conjunctive substructures on the left and disjunctive substructures on the right.
- A unary connective "$*$" to reverse the polarity of a substructure.
- A zero-place item "$\boldsymbol{I}$" to express truth and falsity, depending on the polarity.

The original purpose of display logic was to provide a strategy to prove cut-elimination for relevant logics, where sequents have to be multi-succedent but the presence of the fusion operator makes the usual cut rule for classical logics invalid. Belnap solved the problem by defining "display rules" for structural operators such that the cut formula can always be isolated either in the antecedent or the consequent and a simple cut rule can be applied.

From the expressivist point of view, however, the really interesting result achieved by display logic is the possibility to make explicit the operations on the structure with which formulas occur in an inference, so that rules can be established to govern how such structures can be transformed. Since the meaning of logical operators is grounded in these transformations, the framework is suitable to accommodate various non-classical logics. As a consequence, of course, Belnap's strategy to prove cut-elimination can be extended to several of these.

Some consequences of the approach

There are a few points that it is worth highlighting with respect to these applications of the expressivist strategy for understanding logical forms.

First, in this presentation logical forms are discussed with respect not as much to their impact on proof theory, as to their expressive role. It could be suggestive to imply that the former depends on the latter. Such an implication, however, is not defended in this paper.

Second, the three cases considered here may look like three levels of a cumulative development of the expressive resources of logical languages that improves more and more our understanding of inferences and their structure. However, the acknowledgment of the expressive role of logical forms is distinct from the evaluation of their theoretical function. In fact, the aftermath of the application of the expressivist strategy depends on the theory of inference that is adopted. Depending on the latter, the expressive results delivered by certain logical forms could even be considered as a *reductio* of their use.

Third, the expressivist understanding of logical forms presented here clearly puts most of the burden of the account of logical forms on the theory of inference that defines their nature (cp. Prawitz, 2019). Conflicts about what logical forms are there and what their meaning is are traced back to incompatibilities between different theories of inference. Interestingly, however, such incompatibilities sometimes only emerge thanks to the expressive role of logical forms. In this sense, the expressivist approach could have a positive role to play in the philosophy of logic.

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The Possibilities of Form Logic

Alexandra Zinke (University of Frankfurt, Germany) & Wolfgang Freitag (University of Mannheim, Germany)

In classical first-order predicate logic, and in many other first-order systems, all atomic formulas are essentially of the same logical form: an atomic formula consists of an *n*-ary relation constant and *n* individual terms. This presupposes the classification of terms in exactly two categories: individual terms and relation constants. Taking up a suggestion from Wittgenstein and others, we challenge this assumption and, more generally, the idea that atomic sentences are all of the same logical form.

We develop a *construction schema* for logical systems which allows for an arbitrary number of different kinds of terms and hence a plurality of logical forms of atomic sentences. This is the basic idea of *form logic*. Individual form-logical systems result from different partitions of the terms into form classes and the form-based syntax rules for atomic formulas. Semantically, we treat all terms equally: the *interpretation function* maps each term on an element of the domain. Truth in a model is not determined by the interpretation of the terms alone, but by additional reference to the model's *world*, which is a set of sequences of objects (best understood as atomic facts. An atomic formula is true in a model if and only if its term-wise interpretation is an element of the model's world. Our models thus keep reference and truth apart.

Three form-logical systems are of particular interest and will be briefly presented: The system of *minimal form logic* comes with the most liberal syntax: every finite sequence of non-logical terms is an atomic formula of minimal form logic. The form-logical *reconstruction of first-order predicate logic* partitions the non-logical terms into two form classes and thence delivers syntax rules matching those of predicate logic It shows that predicate logic can be understood as a special case of the general system of form logic. Finally, we introduce the idea of what we deem to be a *Wittgensteinian Begriffsschrift*, the modally adequate form logic. In this system, the terms are partitioned into their 'correct' form classes. This results in a system in which a sequence of non-logical terms is an atomic formula if and only if the proposition it expresses is metaphysically possible, which encodes Wittgenstein's idea that every atomic sentence expresses a possible state of affairs.

We conclude with some remarks about the nature of logical forms and their relevance for the metaphysics of modality.