LOGICA 2022

ABSTRACTS

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Intensionality, invariance, and univalence

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What does a mathematical proposition mean? Under one familiar account, all true mathematical statements mean the same thing, namely *True*. A more meaningful account is provided by the Propositions-As-Types conception of type theory, according to which the meaning of a proposition is the collection of its proofs – i.e. its *means of verification*. The new system of Homotopy Type Theory (HoTT, 2013) provides a further refinement: The meaning of a proposition in HoTT is the *homotopy type* of its proofs. A homotopy type may be represented by an infinite-dimensional structure, consisting of objects, isomorphisms, isomorphisms of isomorphisms, etc. (an ∞-*groupoid*). Such structures occur as systems of objects together with all of their higher symmetries. Now it is a fact that the language of Martin-Löf's intensional type theory is an invariant of all such symmetries, which is enshrined in the celebrated Principle of Univalence (Awodey, 2018).

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Belnapian logics for uncertainty

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Reasoning about information, its potential incompleteness, uncertainty, and contradictoriness need to be dealt with adequately. Separately, these characteristics have been taken into account by various appropriate logical formalisms and (classical) probability theory. While incompleteness and uncertainty are typically accommodated within one formalism, e.g. within various models of imprecise probability, contradictoriness and uncertainty less so — conflict or contradictoriness of information is rather chosen to be resolved than to be reasoned with. To reason with conflicting information, positive and negative support—evidence in favour and evidence against—a statement are quantified separately in the semantics. This two-dimensionality gives rise to logics interpreted over twist-product algebras or bi-lattices, the well known Belnap-Dunn logic of First Degree Entailment being a prominent example (Belnap, 2019; Dunn, 1976). Belnap-Dunn logic with its double-valuation frame semantics can in turn be taken as a base logic for defining various uncertainty measures on de Morgan algebras, e.g. Belnapian (non-standard) probabilities (Klein et al., 2021) or belief functions (Zhou, 2013; Bílková et al., 2022; Frittella et al., 2022).

In spirit similar to Belnap-Dunn logic, we have introduced many-valued logics suitable to reason about such uncertainty measures. They are interpreted over twist-product algebras based on the $[0,1]$ real interval as their standard semantics and can be seen to account for the two-dimensionality of positive and negative component of (the degree of) belief or likelihood based on potentially contradictory information, quantified by an uncertainty measure. The logics presented in this talk include expansions of Łukasiewicz logic with a de-Morgan negation which swaps between the positive and negative semantical component. The resulting logics inherit both (finite) standard completeness properties, and decidability and complexity properties of Łukasiewicz logic, and allow for an efficient reasoning using the constraint tableaux calculi formalism (Bílková et al., 2021).

Two-layered logics for reasoning under uncertainty of classical events (Fagin et al., 1990; Hájek et al., 1995), developed further within an abstract algebraic framework by (Cintula and Noguera, 2014) and (Baldi et al., 2020), separate two layers of reasoning: the inner layer consists of a logic chosen to reason about events or evidence, the connecting modalities are interpreted by a chosen uncertainty measure on propositions of the inner layer, typically a probability or a belief function, and the outer layer consists of a logical framework to reason about probabilities or beliefs. The modalities apply to inner level formulas only, to produce outer level atomic formulas, and they do not nest. Logics introduced in (Fagin et al., 1990) use classical propositional logic on the inner layer, and reasoning with linear inequalities on the outer layer. (Hájek et al., 1995) on the other hand use Łukasiewicz logic on the outer layer, to capture the quantitative reasoning about probabilities within a propositional logical language.

Our main objective is to utilise the apparatus of two-layered logics to formalise reasoning with uncertain information, which itself might be non-classical, i.e., incomplete or contradictory. Many-valued logics with a two-dimensional semantics mentioned above are used on the outer layer to reason about belief, likelihood or certainty based on potentially incomplete or contradictory evidence, building on Belnap-Dunn logic of First Degree Entailment as an inner logic of the underlying evidence. This results in two-layered logics suitable for reasoning scenarios when aggregated evidence yields a Belnapian probability measure (Bílková et al., 2020) or a belief function (on a De Morgan algebra) (Bílková et al., 2022).

This talk is rooted in joint work with S. Frittella, D. Kozhemiachenko, O. Majer and S. Nazari.

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Non-distributive logics: from semantics to meaning

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In this talk, which is based on co-authored work (Conradie et al., 2020), I discuss an ongoing line of research in the relational (non topological) semantics of non-distributive logics. The developments I will discuss are technically rooted in dual characterization results and insights from unified correspondence theory. However, they also have broader, conceptual ramifications for the intuitive meaning of non-distributive logics, which we explore.

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Symmetry principles in inductive logic

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In the case of propositional calculus, the idea of the degree of probability lent by one proposition to another, as proposed by Bernard Bolzano and also by Ludwig Wittgenstein, appears quite natural and almost inevitable, a matter of logic alone. Yet the effort to provide a similar theory for the predicate logic has met with considerable difficulties both in establishing what it is that needs to be done and in developing the theory that delivers it.

We survey how the approach familiar from the propositional context leads to Rudolf Carnap's (pure) inductive logic, and we highlight its aims as stated in Carnap's 1966 paper *The Aim of Inductive Logic*. It involves a search for a logically justifiable prior probability function that could serve as a starting point for inductive reasoning to be carried out by a 'robot' or an 'idealized baby'. This means that it should be carried out by a rational agent who originally has only the language and logic at his or her disposal, and who should develop their probability distribution via repeated conditioning on information as it sequentially comes in.

In the quoted paper, Carnap suggests that the choice of the desired rational prior probability function is to be narrowed by eliminating probability functions that appear irrational because they are not indifferent where there is no reason for differentiating. That is, we should focus on the 'valid core' of Laplace's principle of indifference and accept those instances of it that are intuitively logically justifiable. Somewhat confusingly, modern pure inductive logic often refers to instances of the old principle of indifference summarily as principles of symmetry whereas Carnap speaks about principles of invariance; Carnap's Axiom of Symmetry (today's Constant Exchangeability) is just one of them.

In papers Paris and Vencovská (2011, 2012), a very general symmetry principle INV was proposed. In the former of these papers, it was also shown that INV goes too far and that in the case of unary languages, it eliminates all but one, somewhat unsuitable, prior probability function. Recent research appears to throw some light on what is wrong with such a strong principle, why it should not be accepted, and what should replace it. Again, it has been possible to clarify the effect of the new suggestion completely for unary languages, where it somewhat surprisingly reduces to the well-known and well-studied principle of Atom Exchangeability. We also offer some insights into what might be true in the polyadic case.

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A family of modal fixpoint logics

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Modal fixpoint logics are extensions of basic modal logic, either with fixpoint connectives, such as the common knowledge operator in epistemic logic or the until operator in temporal logic, or with explicit least- and greatest fixpoint operators, as in the modal mu-calculus. Such formalisms may significantly increase the expressive power of the language, by enabling the expression of recursive phenomena.

In the talk I will discuss a small family of modal fixpoint logics that we obtain by syntactically restricting the application of the fixpoint operators in the modal mu-calculus. This family contains some interesting and well-known members, such as propositional dynamic logic and the alternation-free mu-calculus. I will review some recent results on the model theory and the proof theory of this family,

I will assume some rudimentary knowledge of basic modal logic, but no prior acquaintance with the modal mu-calculus.

Remarks on semantic information and logic

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The paper deals with the notion of semantic information carried (or conveyed) by a declarative sentence, especially information carried by a formula in certain propositional languages in a given model in virtue of the meaning of the logical operations. The focus is thus on logical information and not on information in terms of the descriptive content of atomic sentences. If the information carried by a formula A in a model is represented by sets of states at which A is semantically evaluated, then "classically" the evaluation gives rise to a distinction between two sets, the set of states at which A is true, A's truth set in the model, and the set of states at which A is false. The information carried by A is given already with A's truth set (also called "the UCLA proposition expressed by A"), because falsity is identified with untruth and A's truth set determines its complement as A's falsity set. If we shift our attention from truth and falsity to information given with respect to the truth or falsity of atomic formulas and ultimately arbitrary formulas, we are dealing with what Nuel Belnap [1, 2] has called "told values": **T** (*told true but not false*), **F** (*told false but not true*), **N** (*told neither true nor false*), **B** (*told both true and false*). The information carried by a formula A in a model is then represented by four sets of states, and the set of states at which a formula A is told false need not coincide with the set of states at which A fails to be told true.

The states of a model can be seen as *information states* as they represent the semantic information that is given with a valuation function. With Belnap's four-valued functions, a state may support the truth or the falsity of an atomic formula, and if no combination of being told is excluded, there may be states at which a given atomic formula is both told true and told false (states that support both the truth and the falsity of the formula) and states at which the formula is neither being told true nor being told false (states that neither support the truth nor the falsity of the formula). As is well known, the set of states can given a relational or algebraic structure. In Grzegorczyk's [3] and Kripke's [4] informational interpretation of intuitionistic logic, the non-empty set of states is pre-ordered or partially ordered by a binary relation of possible expansion of information states. The semantics is made many-valued in the relational semantics for Nelson's constructive logics with strong negation **N3** and **N4**, see [8] and references therein, by introducing two separate satisfiability relations, verification (support of truth) and falsification (support of falsity). Informationally interpreted algebraic structures for substructural subsystems of intuitionistic logic and Nelson's logics, namely models based on semilattice-ordered monoids, have been studied in [15, 16]. Also in Urquhart's semilattice semantics for relevance logic the set of states has an algebraic structure, featuring a binary operation of combination of information states (or pieces), see also [9], [20]. The ternary relation used in Routley-Meyer models for relevance logic has been given an informational reading by Mares [6, 7] and, more recently, Punčochář and Sedlár have developed an *information based semantics* in the context of inquisitive logic [10], [11].

Whilst the use of such relational and algebraic information structures turned out to be a rich and flexible approach in the study of substructural and other non-classical logics, I will focus on further semantical categories in addition to truth and falsity, respectively support of truth and support of falsity. With the distinction between sense and reference, Gottlob Frege enriched the inventory of basic semantical categories and values. Next to truth and falsity there are meaningfulness and meaninglessness (nonsensicality). Although according to Frege in a scientific language it ought to be the case that the sense of a sentence (the thought expressed by it) determines the sentence's reference (its truth value *The True* or *The False*), Frege nevertheless acknowledged natural language sentences that have a meaning but no reference. The four basic semantic values (*true*, *false*, *meaningful*, and *nonsensical*) induce a set of sixteen told values, including the values *told both meaningful and false* and *told both meaningful and nonsensical*. In this paper I will present two non-classical logics in languages that contain the unary connectives $[m]$ ("it is meaningful that") and $[n]$ ("it is nonsensical that"). One system, **N4mn**, is an expansion of the four-valued constructive and paraconsistent logic **N4**, and it is presented in [19] as a case study in logical tetralateralism. The other system, $\mathbb{I}\mathbb{n}\mathbb{f}$, is a logic interpreted on a 16-element lattice $\mathbb{16}_{inf} = (\mathbb{16}, \subseteq)$ of generalized truth values generated from the set of the four basic semantical values by considering its powerset, $\mathbb{16}$. In **N4mn**, the information carried by a formula A in a model is represented by 16 sets of states, in $\mathbb{I}\mathbb{n}\mathbb{f}$ it is represented by one out of 16 semantical values.

The move from metaphysically understood semantical values to informational told values allows one to take a fresh look at logical consequence and hence on logic. On the standard conception, semantic consequence is understood as truth preservation from the premises to the conclusion of an inference, and, from a "classical" point of view, as untruth preservation from the conclusion to the premises. From the informational point of view, one may think of logic as the study of information flow, see [5], [17], [18]. Information flow, however, comes in more than one flavor depending on the basic semantic categories. In a valid inference, the information that the premises are true, false, meaningful, respectively nonsensical provides the information that the conclusion is true, false, meaningful, respectively nonsensical; that is, if the premises are told true, false, meaningful, respectively nonsensical, then so is the conclusion.

In the paper, the 16-valued logic **N4mn** is introduced semantically and shown to be faithfully embeddable into positive intuitionistic propositional logic. The logic $\mathbb{I}\mathbb{n}\mathbb{f}$ is new. It is introduced as a formula-formula inference system and is shown to be sound and complete with respect to $\mathbb{16}_{inf}$. Semantic consequence is defined with respect to the subset relation as an information order on $\mathbb{16}$, and set intersection (union) as the lattice meet (join) gives rise to a conjunction (disjunction) connective. The presentation ends with the definition of a 65536-element pentalattice, $\mathbb{65536}_5$, with five lattice orderings: an information preorder, a truth preorder, a falsity preorder, a meaningfulness preorder, and a nonsensicality preorder. This step is motivated by the rationale for proceeding from the smallest non-trivial bilattice $FOUR_2$ to the trilattice $SIXTEEN_3$, see [12, 13].

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Contributed talks

‘Bad’ reductions, paradoxes and the meaning of proofs

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This paper is concerned with questions about reductions in natural deduction and λ-calculus, and their relations to questions about meaning of proofs, paradoxes and correspondences in sequent calculus. The reductions for our usual connectives, corresponding to β-reductions in λ-calculus, are meant to eliminate unnecessary detours of the following form: There is a formula, called *maximal formula*, which is both the conclusion of an introduction rule of a connective as well as the major premise of an elimination rule governing the same connective. It can and has been argued, however, that there are more reductions than the ones that are usually considered (see, e.g., Tennant (1995)). One of those, presented in (Ekman, 1994, 1998), is the following:

$$\dfrac{B \to A \qquad \dfrac{A \to B \qquad \begin{matrix}\mathscr{D}\\ \vdots \\ A\end{matrix}}{B}\,{\to}E}{A}\,{\to}E \qquad \rightsquigarrow_{Ekman} \qquad \begin{matrix}\mathscr{D}\\ \vdots\\ A\end{matrix}$$

The motivation to discuss this reduction is related to Tennant’s (1982) proof-theoretic characterization of paradoxes. According to this, paradoxes are to be seen as non-normalizable derivations of $\bot$ in the context of proof theory. They lead from certain paradoxical sentences to a proof of absurdity (or in Curry’s paradox the proof of an arbitrary atomic formula), which is not normalizable. The proofs are in non-normal form and all attempts to apply reduction procedures eventually end up with the original proof. Thus, the normalization sequences for such proofs enter a loop, never ending with a proof in normal form.[1] What Ekman wanted to show with his reduction is that we can get a non-normalizable derivation of $\bot$, which is *not paradoxical* in nature, though, because it does not contain any “paradoxical sentences” like the Liar, for example. Thus, if we would accept this reduction, this would show that this feature of looping non-normalizability of a derivation of $\bot$ is not due to paradoxical sentences or connectives but can occur in a system without these, as well.

However, although Ekman-reduction is certainly a reduction in the sense that there is an elimination of what seems to be an unnecessary detour, I will argue that it is not an acceptable reduction. There have been several different ways to respond to Ekman, e.g. (Schroeder-Heister/Tranchini, 2017), which has in turn spawned further responses such as (Tennant, 2021). Schroeder-Heister and Tranchini argue that Ekman-reduction needs to be rejected because it would lead to a trivialization of identity of proofs in the sense that every derivation of the same conclusion would have to be identified (if reductions are taken to preserve identity of proofs). Tennant, on the other hand, points out that the peculiarity of this reduction resolves once we use what he calls *parallelized* elimination rules instead of the usual *serial* ones (as in the derivation above).[2] He claims that if we use the parallelized elimination rules in the construction of Ekman’s paradox instead, then the derivation can actually be given in normal form, i.e., we do not get into a looping normalization sequence.

[1] As Tennant (1995) later refined: it does not have to be a loop but can also be a non-terminating sequence, as in the case of Yablo’s paradox.

[2] The parallelized elimination rules correspond to what elsewhere, e.g. in (Negri/von Plato, 2001), is called *general* elimination rules.

I will argue, though, that the underlying conception of normal forms of derivations as well as of reductions is misleading and therefore the cause of what seems to be a paradoxical derivation. A comparison with the corresponding steps in sequent calculus can help to clarify the problematic features. My method will be to exploit the Curry-Howard-correspondence (see, e.g., Sørensen/ Urzyczyn (2006)) and look at proof systems annotated with λ-terms. These make the structure of our derivations explicit and allow a much easier way to compare and transfer derivations of natural deduction and sequent calculus. Since both these points are important for my aim, I think it is advantageous to use term-annotated systems. This will allow us to show in a much simpler way what is wrong with potential reductions and why they should not be admitted in our system.

Thus, it can be shown that the redundancies we observe in a natural deduction representation of Ekman's paradox are not present if we transfer it into a derivation in sequent calculus, *although* it is indeed possible to transfer the general Ekman-redundancy and -reduction to sequent calculus. This is connected to our notions of normalization and cut-elimination, which I will therefore argue to treat more carefully in the light of non-standard cases like paradoxical derivations.

On this basis I will give a criterion for the acceptability of reductions which will rule out Ekman-reduction and 'tonkish'-reductions but, on the other hand, leaves room to decide for other non-standard reductions whether or not they should be accepted. Since this criterion will be a type-theoretical property, it can be used for both natural deduction and sequent calculus, and the specific representation of the rules does not play a role, either (as in Tennant's argumentation). The motivation for this criterion will be philosophical, though. I will argue that the question, which reductions we accept in our system, is important for questions of proof identity but, more importantly, is also decisive for the more general question whether a proof has meaningful content. If we accept *any* reductions, we would not only be forced to identify proofs of the same formula (as Schroeder-Heister/Tranchini (2017) showed) but also of arbitrarily different formulas. Even if we reject the herein underlying assumption that reductions induce an identity relation between proofs, I will argue that allowing certain reductions would render derivations in such a system *meaningless*. Therefore, my proposed criterion for the acceptability of reductions ensures the non-triviality of our system both with respect to identity of proofs as well as to their meaning.

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A Vopěnka-style indistinguishability principle for formal fuzzy mathematics

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A general feature of human (and machine) perception is that some objects cannot be distinguished from one another, in one or more aspects. In mathematics, this kind of perceptual indistinguishability can be modelled by various mathematical structures, including binary relations of equivalence or proximity, metrics or topologies, rough sets, and fuzzy similarity relations. Of these, fuzzy similarity relations (Zadeh, 1971; Valverde, 1985) turn out to be a particularly appealing model: First, similarly to metrics, they allow us to express the degrees of distinguishability between objects, or the fact that some pairs of objects can be distinguished more clearly than others (recall that fuzzy relations are represented by binary functions into the interval [0,1] or another suitable scale L). Moreover, they offer a solution to *Poincaré's paradox,* i.e., the fact that indistinguishability should intuitively be transitive, yet in a sufficiently long series where every two neighbouring elements are mutually indistinguishable, the extremal elements may easily be distinguishable from each other (Poincaré, 1902). Fuzzy similarity relations admit the second horn of the paradox while still being fuzzily transitive, i.e., satisfying the usual formula for transitivity if it is semantically interpreted by means of a t-norm fuzzy logic (for t-norm fuzzy logics see, e.g., Hájek, 1998). Via suitable antitonic functions, fuzzy similarity relations are dual to certain classes of generalized metrics; consequently, many topological and metric notions carry over to fuzzy similarity relations, including such concepts as boundedness, dimension, and compactness.

Another intriguing model of indistinguishability is offered by Vopěnka's Alternative Set Theory (AST), which represents it by an equivalence relation that arises from discrimination via infinitely many progressively sharpened perspectives (Vopěnka, 1979). In this contribution, however, I rather want to draw on another fundamental idea of Vopěnka's theory, namely the characterization of finite sets in terms of the surveyability and clear discernibility of all their elements by (possibly enhanced, but still limited) human means. One way of interpreting Vopěnka's principle of finiteness-as-discernibility (or equivalently, infinity-as-indiscernibility) is that in any infinite set, some elements must inevitably be indistinguishable from one another. If we abstract away from the specifics of AST and apply the latter principle to the model of indistinguishability in fuzzy logic, it amounts to the requirement of *precompactness* (i.e., total boundedness) of the generalized metric dual to the fuzzy similarity relation. In particular, the principle postulates that the fuzzy similarity relation $R\colon U^2 \to L$ satisfies the condition $\bigvee_{x,y\in A, x\neq y} Rxy = 1$ for every infinite set $A \subseteq U$. This condition can be expressed in the language of first-order fuzzy logic and investigated by its formal methods.

In the talk, I will show several consequences of the precompactness principle for fuzzy indistinguishability relations. One such consequence is the existence of fuzzy minima in fuzzy orderings congruent with a precompact fuzzy equivalence relation; this fact finds use in the recently proposed fuzzy semantics of counterfactual conditionals (Běhounek and Majer, 2021). Another is the structure of fuzzy neighborhoods of $+\infty$ in the real line generated by precompact order-congruent fuzzy indistinguishability relations, which provides an additional justification of the notion of fuzzy limit (Soylu, 2008) as an alternative foundation of infinitesimal calculus. I will argue that the proposed principle captures the human discernibility limitations in the fuzzy model of indistinguishability of objects, and thus represents an important principle for fuzzy mathematics formalized in t-norm fuzzy logics.

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Logical instrumentalism for anti-exceptionalists

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Anti-exceptionalism about logic is the thesis that logical theories are significantly similar to scientific theories with respect to their epistemic status and methodology (Hjortland 2017). Within that debate, some authors have proposed versions of *logical instrumentalism*, being the idea that logic should essentially be understood as a tool or instrument to achieve particular purposes (Arenhart 2020, Dos Santos 2021, Peregrin and Svoboda 2021). In this paper I will assess logical instrumentalism as it has been put forth in the anti-exceptionalism debate, and will argue that if one wishes to uphold the claim that logic is significantly similar to science, logical instrumentalism cannot be what previous authors have taken it to be. I will put forth a different kind of logical instrumentalism, labelled *representational logical instrumentalism*, that is better aligned with, and informed by, scientific instrumentalism.

Following an early presentation due to Haack (1978), logical instrumentalism can be characterized as the combination of two separate views on logic: the logic-as-tool view and non-representationalism about logic:

- **Logic-as-Tool View**: logical systems are best understood as tools or instruments to achieve particular goals or purposes.
- **Non-Representationalism about Logic**: logics do not represent any extra-systematic phenomenon

Recently, various authors have put forth a version of logical instrumentalism along these lines in the anti-exceptionalism debate. In this paper, I will focus on three of these proposals, namely Arenhart (2020) and Dos Santos (2021), and Peregrin and Svoboda (2021). First, I will show that these proposals can be labelled logical instrumentalist.

In reply to the background logic group, Arenhart (2020) is lead to endorse a form of logical nihilism, but he quickly points out, however, that "[l]ogical nihilism does not mean abandoning the very idea that a system of logic can be chosen for given purposes, and that one of them may be better suited to deal with the evidence than others" (p. 22). This is the logic-as-tool view. Arenhart then endorses non-representationalism when he writes that "we also abandon the idea that the aim of the activity of logicians is attempting to find out something that is already there 'in the wild, the idea that there is a notion of validity simpliciter" (p. 22). That leaves him with accepting only instrumental applications of logical systems. Thus, this proposal, explicitly referred to as a version of anti-exceptionalism, has all the elements of logical instrumentalism.

Dos Santos (2021) criticizes anti-exceptionalist accounts of theory-choice in logic that rely on pre-theoretical logical intuitions for the assessment of candidate logical theories. Dos Santos takes these accounts to aim for an accurate representation of such intuitions, but he argues against the reliability of these intuitions, and he moves on to argue that logical theories are not representational, but rather ameliorative. That is, logical theories do not aim for an accurate representation of intuitions about logical consequence, but rather aim to improve upon such intuitions. Furthermore, the choice of a logical theory, according to Dos Santos "is always instrumental, to fulfil certain investigative purposes in specific contexts (p. 12219-20). Dos Santos points out that on the ameliorative account, there is no matter of fact about whether there are universally true logical laws (p. 12220). As such, he adopts both the logic-as-tool view and non-representationalism about logic.

A final example is due to Peregrin and Svoboda (2021), who aim to put forward and defend a view of the nature of logic they call moderate anti-exceptionalism. They take aim at the idea that the phenomenon logic aims to account for is that of *genuine* validity. By arguing that the genuine logic can neither be an artificial language (for that would already presuppose a notion of validity) nor natural language (for that would make the issue of a genuine logic into empirical linguistics), they argue against the idea that there is something like *genuine* validity. They then move on to argue that "[i]f we give up on the idea of genuine logic we are [...] left with logic as a human project a project launched primarily to assure that our communication can be, whenever it is desirable, subject to public control (p. 8784). On their picture, logic is not descriptive, i.e. non-representational, but logic is primarily a tool or technology to improve communication, i.e. the logic-as-tool view.

I will move on to argue that the *non-representational* instrumentalist positions presented in the above are implausible and insignificant taken by themselves. First of all, the idea that logical *systems* have instrumental value is trivially true and thus entirely uncontroversial: it is simply a fact that different logical systems have been successfully applied to a variety of applications, such as in mathematics, computer science, linguistics, or electronic circuit design (Cook 2010, Priest 2006). Van Benthem (2008) criticizes the view of logic "as an arsenal of formal systems from which an applied logician can choose given any conceivable task at hand (p. 70, fn. 7), for he takes such a view to lead to the problem of *system imprisonment*: a narrowed focus merely on technical problems internal to a particular formal system, without any external counterpart. Logics are not *just* formal systems, the morale appears to be. This leads me to a second point: combined with the logic-as-tool view that emphasises the success of logical systems on a variety of applications, non-representationalism faces the challenge of accounting for precisely the success of a particular logical system. The problem is that if there is nothing that logic latches on to, then it appears to be that we have no way to account for the fact that logical systems can be successfully applied for particular purposes. And so, they might indeed turn out to be useful as a practical tool, but since their success is apparently only due to magic or happenstance, we are offered no significant or philosophically illuminating insights. For example, it cannot account for the fact that arguments of a certain form consistently turn out to preserve truth. Finally, positions that deny that logical theories have metaphysical or extra-systemic content appear to move away from the idea that logic is significantly similar as the (empirical) sciences, and perhaps in effect liken logic more to a prescriptive discipline such as ethics, rather than to the (empirical) sciences (Martin and Hjortland 2022).

The latter points leads me to my second critique of current instrumentalist positions in the anti-exceptionalism debate: I will argue that logical instrumentalism as it has been presented so far in the anti-exceptionalism debate is not in line with instrumentalism as it has been presented in the sciences. As such, current instrumentalism proposals in the anti-exceptionalism debate are in a relevant sense *contrary* to the aims of anti-exceptionalists. As I will aim to show, the challenges for logical instrumentalism that I have presented in the above, are similar to the challenges raised earlier for scientific instrumentalism. I will show how scientific instrumentalism has shown to avoid the pitfalls that I have argued in the above logical instrumentalism faces, by allowing at least some kind of representation. That makes proposing a kind of logical instrumentalism along the lines of scientific instrumentalism an interesting project, particularly for the anti-exceptionalist about logic.

If logical instrumentalism is to be a viable option for the anti-exceptionalist, we need to (i) articulate a version of it that rejects non-representationalism about logic, and (ii) is instrumentalist about *theories* rather than only about *systems*. Such a logical instrumentalism promises to have at least two upshots: first, it would solve the problem of accounting for the success of a logical system, and second, it would make for a closer affinity between logic and science than how current instrumentalist proposals in the anti-exceptionalism debate have it. Informed by the previous discussion of scientific instrumentalism I will propose *representational* logical instrumentalism as the kind of instrumentalism that anti-exceptionalists should endorse: representational logical instrumentalism emphasises the instrumental value of logical *theories*, which are taken to account for particular extra-systematic phenomena.

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Arbitrary abstraction and logicality

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Abstractionist theories are systems composed by a logical theory augmented with one or more abstraction principles (AP), of form: $f_R\alpha = f_R\beta \leftrightarrow R(\alpha,\beta)$ – that introduce, namely rule and implicitly define, the corresponding term-forming operators f_R. Thus, the logicality of these theories plainly depends on the logicality of the abstraction principles. This issue was originally raised into the seminal abstractionist program, Frege's Logicism – proposed with the foundational purpose to derive arithmetical laws as logical theorems and to define arithmetical expressions by logical terms. As is well-known, this project failed, but the issue of logicality represents, still today, an open question of the abstractionist debate (cf. Tarski, 1956; Fine, 2002; Antonelli, 2010; Cook, 2016; Boccuni & Woods, 2020; Ebels-Duggan, 2019). More precisely, given a semantical definition of logicality as permutation and/or isomorphism invariance, we are able to prove that some abstraction principles (like Hume's Principle) are logical (Cook, 2016)[1] but their implicit *definienda* are not (Antonelli, 2010)[2] – so preventing a full achievement of Logicist goal.

My preliminary aim will consists in showing that this unfortunate situation closely depends on the (unjustified) adoption of a same notion of reference for all the expressions of a same syntactical category (e.g. singular terms as always referential and denoting singular, knowable and standard objects). On the contrary, a less demanding reading of the abstractionist vocabulary – namely, a reading that renounces to the semantical assumption mentioned above – is available; furthermore, such a reading, by admitting a different evaluation of primitive an defined expressions, is able to focus on the only information actually provided by the APs and turns out to be preferable because it is more faithful to the theory. Thus, chosen this reading of the APs and, particularly, an arbitrary interpretation (cf. Brekenridge & Magidor, 2012) of the abstractionist vocabulary, my double aim will consist in inquiring its consequences on the logicality of abstractionist theories both from a formal and from a philosophical point of view.

On the one side, from a formal point of view, given such an interpretation of the APs, we can rephrase the main criterion of logicality for abstraction operators (*objectual invariance*, (cf. Antonelli, 2010)), obtaining a weaker one (*general objectual invariance*[3], GWI, (cf. Woods, 2014; Boccuni & Woods, 2020) and proving that it is satisfied not not only by cardinal operator but also by many other second-order ones, including those implicitly defined by consistent weakenings of Fregean Basic Law V. So, we will note that, given (what I argued as) a preferable reading of the APs, both main strategies pursued in the last century to save Fregean project – Neologicism and consistent revisions of *Grundgesetze* – are able to achieve the desirable logicality objective. Further generalising, I will prove that the logicality criterion

[1] More precisely, some abstraction principles (like Hume's Principle) satisfy the criterion of *contextual invariance* and their abstraction relations (e.g. equinumerosity) satisfy many logicality criteria, like *weak invariance*, *internal invariance*, *double internal invariance*. (Cf. Antonelli, 2010; Fine, 2002; Cook, 2016).

[2] More precisely, the corresponding abstraction operators (e.g. cardinal operators) do not satisfy the criterion of *objectual invariance*. Furthermore, such criterion fails precisely in case of operators related to internal (and,*a fortiori* double internal) invariant relation (cf. Antonelli, 2010). So, operators fail to be *logical* though – just in case – they are implicitly defined by *logical* AP.

[3] An expression ϕ is *generally weak invariant* just in case, for all domains D, D' and bijections ι from D to D', the set of candidate denotations of ϕ on D (ϕ^{*D}) = $\{\gamma : \gamma$ is a candidate denotation for ϕ on $D\}$ is such that $\iota(\phi^{*D})$ = $\phi^{*D'}$ = $\{\gamma : \gamma$ is a candidate denotation for ϕ on $D'\}$.

could be satisfied by a large range of APs and is apparently liable to a triviality objection – e.g. it is not able to distinguish between HP and some of its Bad Companions (like Nuisance Principle). I will answer to such a potential objection by showing that GWI however introduces interesting differences. More precisely, I will discuss the controversial case of Ordinal Abstraction and I will prove that GWI is not satisfied by any first-order abstraction principles (cf. Tarski, 1956; Woods, 2014). So, by comparing respective schemas of first-order and second-order APs, we will note that logicality (in the chosen meaning) mirrors a relevant distinction between same-order and different-order abstraction principles.

On the other side, from the philosophical point of view, I will focus on the role of arbitrariness as a condition for the adoption of the abovementioned logicality criterion. Particularly, while this last one seems to testify the unexpected availability of the Logicist goal, the arbitrary interpretation of the vocabulary seems to include semantical insights that are radically alternative to Logicism. In order to argue for this latter consideration, an analogy between the arbitrary interpretation of the APs and the semantics of some eliminative Structuralist reconstructions of the scientific theories (Schiemer & Gratzl, 2016) will be illustrated.

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On duality between partiality and paraconsistency

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Partiality and paraconsistency are metatheoretical properties satisfied by a wide range of non-classical logics that are sometimes very different in nature. This talk is intended to provide a unified understanding of some partial and paraconsistent logics through an investigation of their duality relations (Dunn, 2000; Brunner & Carnielli, 2005; Wansing, 2010). To this end, three steps mark out this talk.

A Four-Valued Extension of $\mathsf{K_tT4}$

The temporal logic $\mathsf{K_tT4}$ is the modal logic obtained from the minimal temporal logic $\mathsf{K_t}$ (Rescher & Urquhart, 1971) by requiring the accessibility relation to be reflexive (which corresponds to the axiom T) and transitive (which corresponds to the axiom 4). This section aims at providing a labelled sequent calculus for an extension of $\mathsf{K_tT4}$ based on Dunn-Belnap's four-valued logic (Belnap, 1977). This many-valued modal logic is here referred to as $\mathsf{K_t^4T4}$.

The *language* of $\mathsf{K_t^4T4}$, denoted by $\mathscr{L}(\mathsf{K_t})$, is composed of a countable set of propositional symbols p_n for every $n \in \mathbb{N}$ plus the propositional logical symbols $\neg$, $\wedge$, $\vee$ and the modal logical symbols $\Box_\mathtt{F}$, $\Diamond_\mathtt{F}$, $\Box_\mathtt{P}$, $\Diamond_\mathtt{P}$ (where 'F' stands for 'future' and 'P' stands for 'past'). The formulae of $\mathscr{L}(\mathsf{K_t})$ are defined as follows.

$$A ::= p \mid \neg A \mid (A \wedge A) \mid (A \vee A) \mid \Box_\mathtt{F} A \mid \Diamond_\mathtt{F} A \mid \Box_\mathtt{P} A \mid \Diamond_\mathtt{P} A$$

The labelled sequent calculus here discussed is based on an internalisation of the relational semantics of $\mathsf{K_tT4}$ into a four-sided sequent calculus closely related to those developed by J.-Y. Girard (Girard, 1976), R. Muskens (Muskens, 1999), and A. Bochman (Bochman, 1998). Similar approaches in the context of two-sided sequent calculi have been discussed, among others, by N. Bonnette and R. Goré (Bonnette & Goré, 1998) as well as S. Negri (Negri, 2005).

A *labelled sequent* Λ is a finite set of labelled formulae and structural elements. A *labelled formula* is a triple $\langle A, \lambda, x\rangle$ where A is a formula of $\mathscr{L}(\mathsf{K_t})$, $\lambda \in \{\mathbf{0}^-, \mathbf{1}^+, \mathbf{0}^+, \mathbf{1}^-\}$ and x is a natural number. A *structural element* is an ordered pair $\langle x, y\rangle$ where x and y are natural numbers.

Intuitively, a labelled formula $\langle A, \lambda, x\rangle$ means: 'Formula A has the minimum degree of falsehood (or, equivalently, is not false) at possible world x' if $\lambda = \mathbf{0}^-$; 'Formula A has the maximum degree of truth (or, equivalently, is true) at possible world x' if $\lambda = \mathbf{1}^+$; 'Formula A has the minimum degree of truth (or, equivalently, is not true) at possible world x' if $\lambda = \mathbf{0}^+$; 'Formula A has the maximum degree of falsehood (or, equivalently, is false) at possible world x' if $\lambda = \mathbf{1}^-$. In the same way, a structural element $\langle x, y\rangle$ means: 'Possible world y is accessible from possible world x'.

Duality

The four-valued modal logic $\mathsf{K_t^4T4}$ satisfies many duality properties. These properties rely on different types of symmetry. In this section, we point out three types of symmetry that are primitive and can be freely combined to define more complex forms of duality. The first two types correspond to an alethic symmetry (either qualitative or quantitative) while the latter corresponds to a relational symmetry. Also, we show properties associated with each of these types of symmetry. In this connection, the dual of a labelled sequent is specified in three different ways.

The first type of symmetry consists in interchanging maximum degree of truth and minimum degree of truth on the one hand and maximum degree of falsehood and minimum degree of falsehood on the other hand. The second type of symmetry consists in interchanging maximum degree of truth and maximum degree of falsehood on the one hand and minimum degree of truth and minimum degree of falsehood on the other hand. The third type of symmetry consists in reversing the accessibility relation (Burgess, 1984).

The *alethic dual* of a formula A of $\mathscr{L}(\mathsf{K_t})$, denoted by $[A]^\dagger$, is defined by induction on the complexity of A as follows.

$$\begin{array}{lcllcl}
[p]^\dagger & = & p & [\Box_{\mathrm{F}} B]^\dagger & = & \Diamond_{\mathrm{F}}[B]^\dagger \\
[\neg B]^\dagger & = & \neg[B]^\dagger & [\Diamond_{\mathrm{F}} B]^\dagger & = & \Box_{\mathrm{F}}[B]^\dagger \\
[(B \wedge C)]^\dagger & = & ([B]^\dagger \vee [C]^\dagger) & [\Box_{\mathrm{P}} B]^\dagger & = & \Diamond_{\mathrm{P}}[B]^\dagger \\
[(B \vee C)]^\dagger & = & ([B]^\dagger \wedge [C]^\dagger) & [\Diamond_{\mathrm{P}} B]^\dagger & = & \Box_{\mathrm{P}}[B]^\dagger
\end{array}$$

The *qualitative alethic dual* of a labelled sequent Λ, denoted by $[\Lambda]^{QL}$, is the set: $\{\langle [A]^\dagger, \overline{\lambda}, x\rangle | \langle A, \lambda, x\rangle \in \Lambda\} \cup \{\langle x,y\rangle | \langle x,y\rangle \in \Lambda\}$ where $\overline{\lambda}$ is defined as follows.

$$\overline{\lambda} = \begin{cases} \mathbf{0}^+ & \text{if } \lambda = \mathbf{0}^- \\ \mathbf{1}^- & \text{if } \lambda = \mathbf{1}^+ \\ \mathbf{0}^- & \text{if } \lambda = \mathbf{0}^+ \\ \mathbf{1}^+ & \text{if } \lambda = \mathbf{1}^- \end{cases}$$

The *quantitative alethic dual* of a labelled sequent Λ, denoted by $[\Lambda]^{QT}$, is the set: $\{\langle [A]^\dagger, \overline{\lambda}, x\rangle | \langle A, \lambda, x\rangle \in \Lambda\} \cup \{\langle x,y\rangle | \langle x,y\rangle \in \Lambda\}$ where $\overline{\lambda}$ is defined as follows.

$$\overline{\lambda} = \begin{cases} \mathbf{1}^- & \text{if } \lambda = \mathbf{0}^- \\ \mathbf{0}^+ & \text{if } \lambda = \mathbf{1}^+ \\ \mathbf{1}^+ & \text{if } \lambda = \mathbf{0}^+ \\ \mathbf{0}^- & \text{if } \lambda = \mathbf{1}^- \end{cases}$$

It is worth noting that the combinaison of the qualitative alethic duality and the quantitative alethic duality gives rise to a form of duality pointed out by J. M. Dunn in the context of ‘first degree entailment’ and based on a symmetry between truth and non-falsehood on the one hand and falsehood and non-truth on the other hand (Dunn, 1976).

The *relational dual* of a formula A of $\mathscr{L}(\mathsf{K_t})$, denoted by $[A]^\ddagger$, is defined by induction on the complexity of A as follows.

$$\begin{array}{lcllcl}
[p]^\ddagger & = & p & [\Box_{\mathrm{F}} B]^\ddagger & = & \Box_{\mathrm{P}}[B]^\ddagger \\
[\neg B]^\ddagger & = & \neg[B]^\ddagger & [\Diamond_{\mathrm{F}} B]^\ddagger & = & \Diamond_{\mathrm{P}}[B]^\ddagger \\
[(B \wedge C)]^\ddagger & = & ([B]^\ddagger \wedge [C]^\ddagger) & [\Box_{\mathrm{P}} B]^\ddagger & = & \Box_{\mathrm{F}}[B]^\ddagger \\
[(B \vee C)]^\ddagger & = & ([B]^\ddagger \vee [C]^\ddagger) & [\Diamond_{\mathrm{P}} B]^\ddagger & = & \Diamond_{\mathrm{F}}[B]^\ddagger
\end{array}$$

The *relational dual* of a labelled sequent Λ, denoted by $[\Lambda]^R$, is the set: $\{\langle [A]^\ddagger, \lambda, x\rangle | \langle A, \lambda, x\rangle \in \Lambda\} \cup \{\langle x,y\rangle | \langle y,x\rangle \in \Lambda\}$.

Unifying Partiality and Paraconsistency

Several well-known partial and paraconsistent logics can be faithfully embedded into $\mathsf{K}_\mathsf{t}^4\mathsf{T4}$. In this connection, three many-valued logics and three constructive logics are addressed. Among the many-valued logics, we consider Kleene's strong three-valued logic, Priest's logic of paradox, and Dunn-Belnap's four-valued logic (Priest, 2008). Among the constructive logics, we investigate intuitionistic logic, dual-intuitionistic logic, and bi-intuitionistic logic (Rauszer, 1980).

Thanks to their embedding into $\mathsf{K}_\mathsf{t}^4\mathsf{T4}$ (Łukowski, 1996), this section intends to propose a unified view of these logics through a general principle of duality. In this regard, a form of duality resulting from the combination of the quantitative alethic duality and the relational duality is identified and shown to apply to every partial or paraconsistent logic discussed. Through this general principle, a perfect symmetry is observed between Kleene's strong three-valued logic and Priest's logic of paradox on the one hand and intuitionistic logic and dual-intuitionistic logic on the other hand. In addition, Dunn-Belnap's four-valued logic, just like bi-intuitionistic logic, is its own counterpart.

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Truthmaker semantics for degreeism of vagueness

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Philosophers have been long discussed vagueness and its related paradox — the sorites — for many reasons, mostly linguistic, sometimes metaphysical, and more (cf. Keefe & Smith (1997)). When philosophers talk about vagueness, they often end up talking about semantics. In fact, many solutions to the paradox suggest revising semantics: supervaluationism renovates semantics with a new formal concept of supervalues, degreeism suggests many-valued logic and its coresponding semantics, and epistemicism suggests keeping classical logic and semantics fixed, but ascribes vagueness to an epistemic issue. In the market of semantic builders, *truthmaker* is a rising star with its expressive power powerful enough for many including hyperintensionality (Fine (2017)).

Still, few have employed truthmakers for vagueness. An exceptional case Sorensen (2001) suggests an argument appealing to truthmaker gaps but only for his particular version of epistemicism. But might there be other applications of truthmaker semantics in the study of vagueness?

The goal of this paper is to offer an affirmative answer to this question, by designing a truthmaker semantics for another position on vagueness. Among several positions, this paper works on a popular one: *degreeism* (*degree theory*). True to its name, degreeism revises the semantic concept of a truth value from binary one (truth 1 and false 0 and nothing else) to a many-valued one (often infinite). However, importing truthmakers into degreeism is not straightforward. While truthmakers are about *quality* and use mereological part-whole relations in their formalization, degreeism is based on a more *quantitative* idea, namely a segment of real numbers $[0,1] \subseteq \mathbb{R}$. But how can we translate mereological structures of truthmaking into degreeists' real numbers and the other way around?

One option is to import *measure theory* (cf. Doob (1994), Capiński & Kopp (1965)). A measure is, very roughly put, a mathematical generalization of geometrical measures such as distance, length, area, and volume. This formal notion of "measuring" the size of a given set is applied to many things such as physical mass and, most importantly for degreeism, probability of events. Given that degreeism is often associated with probability theory as they both feature a fragment of real numbers $[0,1]$ as a central part of their formalization, the match already seems apt. We see an evaluation function μ which assigns a truth value for a given truthmaker as a measure function, which satisfies standard axioms of measure theory.

A glance at the definitions shows how naturally these concepts fit degreeism. For one thing, degreeism stipulates that the measure of the null set is zero

$$\mu(\emptyset) = 0.$$

This seems to correspond to our intuitive idea that if a proposition φ has no truthmaker at all (in other words, nothing in a world supports φ) its truth value should be zero. Also, another important definition of *(countable) additivity* confirms our idea of the relationship between truthmakers and truth values — the more truthmakers (e.g. evidence) a proposition has, the more likely it is that the proposition is true.

Still, measure theory is built upon a set-theoretical setting (i.e. upon families of sets), which is different from the mereological structures of truthmakers. So some formal work is needed in order to offer a mereological version of a measure function μ_{TM}. In this paper, I offer a truthmaker semantics for degreeism of the following form:

$$M = \langle S, \sqsubseteq, \mu_{TM} \rangle,$$

where S is a non-empty set of states (truthmakers), $\sqsubseteq$ is a partial order on S expressing mereological relation for part-wholehoodness, and μ_{TM} is a degreeism evaluation function from propositions to $[0,1] \subseteq \mathbb{R}$, assigning a real number from 0 to 1 to a given propositional letter. The first two suggestions follow from the standard formalism of truthmaker semantics. The last one is original. In particular, I design the following two properties for μ_{TM} so as to behave as a measure. First, we need something corresponding to the null set. In the original measure theory, we have $\emptyset$ as an obvious and natural example. But truthmakers do not have apparent counterparts of the empty set. Second, we need a truthmaker version of additivity. I provide several operations such as $\sqcap$ ($s \sqcap t$ is the overlapping part of s and t) and difference $\setminus$ of truthmakers for this purpose.

Having introduced truthmaker semantics for degreeism, this paper discusses the benefits of this semantics to further support how truthmakers are useful in discussions of vagueness, at least for degreeism. The resulting semantics resolves two formal issues for degreeism. One is about triviality Smith (2008). Some may want to characterize vague predicates (from non-vague ones) by the formal concept of continuity. For instance, we may want to characterize vague terms by whether their evaluation function from (a subset of) $\mathbb{N}$ (e.g. the number of hairs) to truth values $[0,1]$. Unfortunately, this does not work because the domain (the number of hairs, with the most natural topology) is discrete, hence any function from $\mathbb{N}$ is trivially continuous. In my suggested framework, such a worry disappears. The domain is not the natural numbers but a set of truthmakers, whose topology is not necessarily discrete. The other is called the problem of "penumbral connection" Fine (2020). This problem is about how to calculate the truth values of two vague clauses connected by logical connectives. For instance, what happens if two indefinite clauses (i.e. borderline cases) are connected with a conjunction, say, "This ball is purple and this ball is red"? The truth value of this sentence should be zero i.e. definitely false because one ball cannot have different colors at the same time. But typical degreeists say it is indefinite, i.e. somewhere between $[0,1]$. Truthmakers provides an easy way out, though. It is clear that a truthmaker for being red and another truthmaker for being purple are simply incompatible.

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First-order relevant reasoners in classical worlds: A first approach

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(Sedlar & Vigiani, forthcoming) have developed an approach to propositional epistemic logics wherein (i) an agent's beliefs are closed under relevant implication, and (ii) the agent is located in a classical possible world (i.e. the non-modal fragment is classical). The approach they developed is to add or identify classical possible worlds as states in the ternary relational semantics of relevant logics, to define truth and validity at these possible worlds, and to bridge the validities of the underlying relevant epistemic logic into the classical logic using a modal operator. I have constructed first-order extensions of these logics by modifying the framework for quantified modal relevant logics of Ferenz (Ferenz, 2021, 2019), which combines the general frame approaches of Seki (Seki, 2003a,b) (for modal relevant logics) with a generalization of Mares and Goldblatt (Mares & Goldblatt, 2006; Goldblatt, 2011) semantics for the quantified extensions of the relevant logic **R**.

The Mares-Goldblatt interpretation of the quantifiers uses general frames to offer a non-Tarskian truth condition. The Tarskian interpretation is defined such that the truth set of a universally quantified formula is the generalized intersection of the truth sets of each of its instances. (That is, relative to a variable assignment.) The Mares-Goldblatt models do not require the closure of admissible propositions under generalized intersections, and an inferentially suitable conjunction operator is defined to model universally quantified formulas with admissible propositions.

The main formal results of the paper are general and modular soundness and completeness results for a wide range of epistemic logics obtained by strengthening the underlying relevant epistemic logic. The base epistemic relevant logic is the regular (in the (Segerberg, 1971) sense of monotonicity and conjunctive regularity of the modal/epistemic operator) modal extension of the relevant logic **QBM**. The logic **QBM** adds fairly weak quantificational axioms and rules to the relevant logic **BM**, which itself is a kind of minimal logic with respect to the ternary relational semantics.

Part of the project initiated by Sedlar (Sedlar, 2015) and Vigiani (Sedlar & Vigiani, forthcoming) is to identity (rather, to add) robust *possible worlds* in the ternary relational frames. To accomplish this, the behaviour of intensional connectives becomes extensional (i.e., truth-functional). Here, I consider the intensional truth constant **t**, the intensional conjunction (a.k.a. fusion) $\circ$, and the left-implication $\leftarrow$. Additional properties of the models are assumed to render these extensional, and these properties are straightforward generalizations of Sedlar and Vigiani. However, the quantifiers bring additional philosophical debates to the fore. In particular, what exactly is the classical behaviour of a quantifier? At least, that is, with respect to a possible world as opposed to the situations employed in interpreting the ternary relational semantics.

The Mares-Goldblatt interpretation of the quantifier coheres with a distinction often attributed to Russell as an objection to the generalized conjunction interpretation of the universal quantifier. One may well know of each object c_i that it is an F. That is, one might know the (infinite) conjunction of the form $Fc_1 \wedge \dots \wedge Fc_n \wedge \dots$, but not have the additional, and indeed separate, information that the list of objects in that conjunction exhausts all objects. One can know of each chair that is is blue without knowing that all chairs are blue, because one does not have the additional information that *those chairs are all the chairs*. However, along the lines of (Mares, 2009), we might think that a robust possible world does always contain the 'that's all the things' data. Therefore, we might think that possible worlds should employ the generalized intersection interpretation: the Tarskian interpretation of the quantifiers.

Unfortunately, the incompleteness results for quantified relevant logics with the Tarskian interpretation shown by (Fine, 1989) are just around the corner. *We cannot adopt the Tarskian conditions and simultaneously give up general frames.* At least, that is, for strong underlying relevant logics such as **R**. However, we may be able to adopt Tarskian truth conditions while keeping general frames. I will further detail a formal analogy with incompleteness results in quantified modal classical logics, and show the progress made on a route to regaining the Tarskian truth conditions.

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State-sensitive intensional subject-matter and topic sufficiency

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Despite the traditional slogan that logic is "topic-neutral," treating the semantic feature of subject-matter as a logically significant feature has a long history. A particularly natural topic-theoretical thesis in logic is that validity requires that the topic of the antecedent *includes* that of the consequent.

The most well-known implementation of this intuition is William Parry's logic PAI of *analytic implication* of Parry (1933), which takes topic inclusion to be a critical element of analyticity. A more recent logical framework in which the inclusion of subject-matter is taken seriously is the theory of *topic-sensitive intentional modals* (TSIMs) developed and championed by Francesco Berto and his collaborators in *e.g.* Berto (2019) that pair variably strict conditionals with a topic inclusion filter.

Both the semantics for PAI and TSIMs are insensitive to determination of subject-matter of *intensional formulae*. *E.g.*, where $\varphi \to \psi$ is an intensional conditional, the topic of $\varphi \to \psi$ is either just the fusion of the topics of its subformulae or is left undefined. Recent work has introduced *conditional-agnostic analytic implication* CA/PAI, in which an additional degree of control over the topic of intensional operators is made possible by allowing the conditional itself to influence a formulae's overall topic.

Definition 1. *A* CA/PAI *Fine model is a tuple* $\langle W, R, \mathscr{T}, \oplus, f^{\to}, v, t\rangle$ *such that*

- $\langle W, R\rangle$ *is an* S4 *Kripke frame*
- *For each* $w \in W$, $\langle \mathscr{T}_w, \oplus_w\rangle$ *is a join semilattice of topics*
- v *is a valuation from atomic formulae to* W
- *For each* $w \in W$, t_w *is a function mapping atomic formulae to* $\mathscr{T}_w$
- *For each* $w \in W$, $f_w^{\to}$ *is a binary function from* $\mathscr{T}_w \times \mathscr{T}_w \to \mathscr{T}_w$ *such that:*
  - $t_w(\neg\varphi) = t_w(\varphi)$
  - $t_w(\varphi \circ \psi) = t_w(\varphi) \oplus_w t_w(\psi)$ *for extensional connectives* $\circ$
  - $t_w(\varphi \to \psi) = f_w^{\to}(t_w(\varphi), t_w(\psi))$

In many ways, the framework improves upon the coarser models introduced by Fine's Fine (1986), whose analysis is in a sense *maximally coarse*; it renders inert any potential influence intensional conditionals bear on determining subject-matter.

In contrast, Definition 1 presupposes of $f^{\to}$ no property but its mere functionality. The open-ended nature of the definition allows the imposition of fine-grained conditions on the value of $f^{\to}(a,b)$ for arguments a and b. Any number of natural and subtle conditions can be employed to fit the features of particular intensional conditionals.

But for all the modularity and corresponding nuance won by Definition 1, there seem to be natural constraints that cannot be reflected with new conditions on $f^{\to}$. The refined framework encapsulates—for better or for worse—the following thesis: *The topic of an intensional conditional is a function of the subject-matters of its antecedent and consequent.* As this thesis stipulates that the topics of the parts are *sufficient* to determine the topic of the complex, we formally introduce this as *Topic Sufficiency*:

- **Topic Sufficiency:** For an intensional conditional $\rightarrow$, if $t(\varphi) = t(\xi)$ and $t(\psi) = t(\zeta)$, then $t(\varphi \rightarrow \psi) = t(\xi \rightarrow \zeta)$.

There is some phenomenological evidence against this principle, however. Let us give an examination of the thesis of *Topic Sufficiency* and discuss how to generalize to a semantic framework that is able to more faithfully represent the corner-cases.

First, consider counterevidence illustrated by the following scenario: A team of coworkers is aware that a colleague, John, will likely soon resign in favor of a position at a different organization. Several of the team members have a meeting to prepare for this contingency. Now consider a question to the team [Q] and two responses [R1] and [R2]:

[Q] 'What steps should we take if John resigns from his position?'

[R1] 'Should John resign, we will have to find a replacement.'

[R2] 'Should John *not* resign, we will not have to find a replacement.'

According to *Negation Transparency*—that the topics a formula and its negation are identical—the subject-matters of the antecedents of [R1] and [R2] are identical and *mutatis mutandis* for the consequents. Thus, *Topic Sufficiency* predicts that the subject-matters of [R1] and [R2] coincide.

However, the act of asking [Q] has identified the scope of the following discussion as *contingencies in which John resigns*. Insofar as [R1] describes a recommendation that is *responsive to* or *conditioned on* these contingencies, [R1] remains within the boundaries of the discussion, *i.e.*, [R1] is *on-topic*. In contrast, [R2] *fails* to address the contingencies at the heart of the discussion, *i.e.*, one would *reject* [R2] as *off-topic*. But the prediction that the topics of [R1] and [R2] are identical requires that one is off-topic precisely when the other is.

Then we can address the objection and propose an even more refined model for a *state-sensitive* logic S/PAI by making two small modifications to Definition 1. Let $[\![\varphi]\!]_w$ be the set $\{w' \in W \mid wRw' \text{ and } w' \Vdash \varphi\}$. Then:

Definition 2. *A* state-sensitive S/PAI *model is a tuple* $\langle W, R, \mathscr{T}, \oplus, t, f^{\rightarrow}, \Vdash \rangle$ *retaining everything from Definition 1 except:*

- $f^{\rightarrow} : \mathscr{T} \times \mathscr{T} \times W \rightarrow \mathscr{T}$
- $t_w(\varphi \rightarrow \psi) = f^{\rightarrow}(t_w(\varphi), t_w(\psi), [\![\varphi]\!]_w)$

We can make several observations concerning the properties of S/PAI. First, it preserves a number of intuitively correct validities:

Proposition 1. *The formulae* $((\varphi \wedge \psi) \rightarrow \xi) \rightarrow ((\psi \wedge \varphi) \rightarrow \xi)$ *and* $(\varphi \rightarrow (\psi \vee \xi)) \rightarrow (\varphi \rightarrow (\xi \vee \psi))$ *are valid in* S/PAI *models:*

In other words, applying commutation—or distribution or DeMorgan's laws—to the antecedent or consequent of an intensional conditional is topic-preserving.

Yet there are some surprising *invalidities* that arise, as well.

Proposition 2. $(\varphi \vee \psi \rightarrow \xi) \rightarrow (\varphi \wedge \psi \rightarrow \xi)$ *is not valid in* S/PAI *models.*

Intuitively, this fails because in case $[\![\varphi \vee \psi]\!] \neq [\![\varphi \wedge \psi]\!]$, the collections of states making up the topics of $\varphi \vee \psi \rightarrow \xi$ and $\varphi \wedge \psi \rightarrow \xi$ may differ. But because $[\![\varphi \vee \psi]\!] \supseteq [\![\varphi \wedge \psi]\!]$, the topic of the former should in general *include* that of the latter, which leads to the following:

Proposition 3. $(\varphi \vee \psi \rightarrow \xi) \rightarrow (\varphi \wedge \psi \rightarrow \xi)$ *is valid in* S/PAI *models satisfying:*

$$f^{\rightarrow}(a, b, X) \leq f^{\rightarrow}(a, b, Y) \text{ if } X \subseteq Y$$

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A lightweight logic of agency

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One of the most prominent accounts of agency is Belnap and Horty's logical analysis of 'agent i sees to it that proposition ϕ is true' in terms of 'branching time + agent choice' models (BT+AC models) (Horty and Belnap, 1995; Horty, 2001). At the core of this so-called 'stit logic' are the Chellas stit operator, expressing the ability of a group of agents to ensure that a given formula is true regardless of the choices of other agents, and the deliberative stit operator, adding that truth of this formula is not necessary. This framework is very expressive and was put to work to account for other action-based concepts such as obligation (Horty, 2001; Broersen, 2011), influence (Lorini and Sartor, 2016), social commitment (Lorini, 2013) and responsibility (Lorini et al., 2014). Unfortunately reasoning turned out to be hard: deciding satisfiability of formulas involving the Chellas or the deliberative stit operator is NEXPTIME-complete even without temporal modalities as soon as there is more than one agent (Balbiani et al., 2008); it becomes 2EXPTIME-complete with temporal modalities 'next' and 'until' (Boudou and Lorini, 2018); and it is undecidable as soon as agency of sets of agents (groups) comes into play, and so again already without temporal modalities (Herzig and Schwarzentruber, 2008). On the other hand, model checking—which is often considered to be an interesting alternative to satisfiability checking (Halpern and Vardi, 1991)—is basically unfeasible because BT+AC models are typically infinite.

Given these results, it is natural to search for fragments of the language of stit logic where the reasoning problems are simpler. Several such fragments were investigated in (Schwarzentruber, 2012). We here investigate another fragment that is inspired by the investigation of fragments of description logics with simpler complexity. In analogy with that research program we call our logic a lightweight logic of agency.

The basic idea is to encode BT+AC models in a finite and compact way, so that model checking becomes practically feasible. We do so by grounding the basic concepts of stit logics, namely histories and choices, on the concepts of control and attempt. The former is borrowed from (van der Hoek et al., 2011; Herzig et al., 2011) while the latter is borrowed from (Lorini and Herzig, 2008). Both relate agents and propositional variables: when an agent i controls a propositional variable p then she is able to determine the truth value of p at the next state; if this is the case and i moreover attempts to change p then the truth value of p gets flipped at the next state; and if nobody is able and attempting to change the truth value of p then it remains unchanged.

We start from the lightweight version of Pauly's Coalition Logic (Pauly, 2002) that was proposed in (van der Hoek et al., 2011). The semantics of their Coalition Logic of Propositional Control is in terms of a valuation of classical propositional logic (that is, a set of propositional variables) together with a function associating to each propositional variable the agent controlling it. We generalise this by associating to each variable the (possibly empty) set of agents controlling it. We moreover add to these models a function associating to each propositional variable the set of agents attempting to change it. States are therefore triples, and each triple determines a unique next state: all those variables whose change is attempted by some agent controlling it get their truth value flipped, and the other variables keep their truth value. Attempts are naturally viewed as persistent goals: agents abandon the attempt to change p once p has been successfully changed (possibly by somebody else).

Representing control and attempt in this way only allows for reasoning about the next state: as control does not change, attempts that fail at the first step will always fail, and new attempts cannot appear from one state to the next. We go beyond this simple model and introduce *higher-order control and attempt*: for two (possibly identical) agents i and j, i may control whether j controls a propositional variable p; i may control whether j attempts to change p; i may attempt to change whether j controls p; and i may attempt to change whether j attempts to change p. This allows us to reason about future states beyond the next state: we can define meaningful temporal operators of Linear-time Temporal Logic (LTL). We show that our models correspond to particular BT+AC models of standard stit logic, allowing us to interpret modal operators of agency of the kind "group of agents J achieves ϕ"; more precisely, we define a Chellas stit operator and a deliberative stit operator. Beyond the complexity results for model checking of (Herzig et al., 2022) we provide results for validity and satisfiability checking.

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Interpolation for non-normal modal and conditional logics

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In this talk, via a proof-theoretic method, we show that the non-normal modal logics E, M, EN, MN, MC, K, and their conditional versions, CE, CM, CEN, CMN, CMC, CK, in addition to CKID enjoy the uniform Lyndon interpolation property. This result in particular implies that these logics have uniform interpolation. Although for some of them the latter is known, the fact that they have uniform Lyndon interpolation is new. Also, the proof-theoretic proofs of these facts are new, as well as the constructive way to explicitly compute the interpolants that they provide. On the negative side, we show that the logics CKCEM and CKCEMID enjoy uniform interpolation but not uniform Lyndon interpolation. Moreover, we prove that the non-normal modal logics EC and ECN and their conditional versions, CEC and CECN, do not have Craig interpolation, and whence no uniform (Lyndon) interpolation.

In the rest of this abstract, we will discuss the details of the results. Set $\mathscr{L}_\Box = \{\wedge, \vee, \to, \bot, \Box\}$ as the language of modal logics and $\mathscr{L}_\rhd = \{\wedge, \vee, \to, \bot, \rhd\}$ as the language of conditional logics. The sets of positive and negative variables of a formula $\varphi \in \mathscr{L}$, denoted respectively by $V^+(\varphi)$ and $V^-(\varphi)$ are defined recursively as expected. Note that $V^+(\varphi \rhd \psi) = V^-(\varphi) \cup V^+(\psi)$ and $V^-(\varphi \rhd \psi) = V^+(\varphi) \cup V^-(\psi)$, for $\mathscr{L} = \mathscr{L}_\rhd$. Define $V(\varphi) = V^+(\varphi) \cup V^-(\varphi)$. Lyndon interpolation (LIP) and Craig interpolation property (CIP) for logics are defined as usual. In the following, we define Uniform Lyndon interpolation (ULIP) and uniform interpolation (UIP) for logics.

Definition 1. A logic L has *ULIP* if for any formula $\varphi \in \mathscr{L}$, atom p, and $\circ \in \{+, -\}$, there are p°-free formulas, $\forall^\circ p \varphi$ and $\exists^\circ p \varphi$, such that $V^\dagger(\exists^\circ p \varphi) \subseteq V^\dagger(\varphi)$, $V^\dagger(\forall^\circ p \varphi) \subseteq V^\dagger(\varphi)$, for any $\dagger \in \{+, -\}$, and $L \vdash \forall^\circ p \varphi \to \varphi$ and $L \vdash \varphi \to \exists^\circ p \varphi$. Moreover, for any p°-free formula ψ if $L \vdash \psi \to \varphi$, then $L \vdash \psi \to \forall^\circ p \varphi$, and $L \vdash \exists^\circ p \varphi \to \psi$. A logic has *UIP* if it has all the mentioned properties, omitting $\circ, \dagger \in \{+, -\}$, everywhere.

The logic E is defined as the smallest set of formulas in $\mathscr{L}_\Box$ containing classical tautologies and closed under modes ponens and the rule $\dfrac{\varphi \leftrightarrow \psi}{\Box\varphi \leftrightarrow \Box\psi}\ (E)$. Other non-normal logics can be defined by adding the following modal axioms to E:

$$\Box(\varphi \wedge \psi) \to \Box\varphi \wedge \Box\psi \quad \text{(M)}, \quad \Box\varphi \wedge \Box\psi \to \Box(\varphi \wedge \psi) \quad \text{(C)}, \quad \Box\top \quad \text{(N)}.$$

We consider the following non-normal modal logics: $\mathsf{EN} = \mathsf{E} + (N), \mathsf{M} = \mathsf{E} + (M), \mathsf{MN} = \mathsf{M} + (N), \mathsf{MC} = \mathsf{M} + (C), \mathsf{K} = \mathsf{MC} + (N), \mathsf{EC} = \mathsf{E} + (C)$, and $\mathsf{ECN} = \mathsf{EC} + (N)$. Similarly, for conditional logics, CE is defined as the smallest set of formulas in $\mathscr{L}_\rhd$ containing classical tautologies and closed under modes ponens and $\dfrac{\varphi_0 \leftrightarrow \varphi_1 \qquad \psi_0 \leftrightarrow \psi_1}{\varphi_0 \rhd \psi_0 \to \varphi_1 \rhd \psi_1}\ (CE)$. The other conditional logics are defined by adding the following conditional axioms to CE:

$$(\varphi \rhd \psi \wedge \theta) \to (\varphi \rhd \psi) \wedge (\varphi \rhd \theta) \quad \text{(CM)}, \quad (\varphi \rhd \psi) \wedge (\varphi \rhd \theta) \to (\varphi \rhd \psi \wedge \theta) \quad \text{(CC)},$$

$$\varphi \rhd \top \quad \text{(CN)}, \quad (\varphi \rhd \psi) \vee (\varphi \rhd \neg\psi) \quad \text{(CEM)}, \quad \varphi \rhd \varphi \quad \text{(ID)}.$$

We consider the following conditional logics: $\mathsf{CEN} = \mathsf{CE} + (CN), \mathsf{CM} = \mathsf{CE} + (CM), \mathsf{CMN} = \mathsf{CM} + (CN), \mathsf{CMC} = \mathsf{CM} + (CC), \mathsf{CK} = \mathsf{CMC} + (CN), \mathsf{CEC} = \mathsf{CE} + (CC), \mathsf{CECN} = \mathsf{CEC} + (CN), \mathsf{CKID} = \mathsf{CK} + (ID), \mathsf{CKCEM} = \mathsf{CK} + (CEM)$, and $\mathsf{CKCEMID} = \mathsf{CKCEM} + (ID)$.

Theorem 2. (*ULIP*) *The logics* E, M, MC, EN, MN, K*, their conditional versions* CE, CM, CMC, CEN, CMN, CK*, and the conditional logic* CKID *have ULIP and hence UIP and LIP.*

(*UIP*) *The logics* CKCEM *and* CKCEMID *enjoy UIP and hence CIP.*

(*Negative*) *The logics* EC *and* ECN *and their conditional versions* CEC *and* CECN *do not have CIP. As a consequence, they do not have UIP or ULIP. Moreover, the logics* CKCEM *and* CKCEMID *do not enjoy ULIP.*

Proof sketch. To show our result, we use the sequent calculi for these logics. For modal logics the sequent calculi are defined in (Orlandelli, 2020) and the cut elimination theorem is proved. For conditional logics, we introduce the sequent calculi and prove that the cut rule can be eliminated (the sequent calculi for the logics CK, CKID, CKCEM, and CKCEMID were studied in (Pattinson and Schröder, 2009)). To prove ULIP for these logics we extend the notion to sequent calculi. It is easy to see that ULIP for a sequent calculus implies that the corresponding logic has ULIP. Then, using the natural notion of weight on formulas and sequents we can define a well-ordering on the sequents. We use this well-ordering to define the uniform interpolants and prove the desired properties by induction on this well-ordering.

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Modality and the structure of assertion

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Kant writes in the *Critique of Pure Reason* that the modality of a judgement "contributes nothing to the content of the judgement" in question, but concerns rather the relation between this content and "thought as such". However one interprets this claim, the conception of modality it assumes must appear quite different from that implicit in modern modal logic, where modalities do contribute to content. Just as the connectives, the modalities of modern modal logic are propositional operators used in the construction of propositions, and just as them, they are provided with their own clauses in formal semantics. In this talk I wish to clarify the relation between a conception of modality along Kantian lines and that of modern modal logic. More specifically, drawing upon some lectures by Per Martin-Löf on the structure of assertion, I will offer a small development of the syntax for modal logic given by Pfenning and Davies (2001). The development adds conceptual superstructure, but is conservative with respect to the underlying propositional modal logic (S4).

In order to make sense of Kant's discussion of modality one needs a distinction between judgement and the content of a judgement. The syntax of standard propositional and predicate logic does not make such a distinction. One and the same class of symbols—the well-formed formulas—serve to stand for both contents and judgements. For instance, under the intuitive reading of an implicational formula, $A \supset B$, the antecedent, A, is clearly not asserted, hence it can only be a content. If A is the conclusion of a closed natural-deduction derivation, however, it is, under an intuitive reading, asserted as a theorem, hence a judgement.

Martin-Löf type theory differs from most systems of logic in recognizing a distinction between judgement and proposition (Martin-Löf, 1984). The connectives operate on propositions, and the quantifiers on propositional functions. A judgement asserts that a certain proposition, A, is true, often written A true, or that a certain object, a, is a truthmaker of a proposition, A, often written $a : A$. I will concentrate here on the form A true, but a similar development can be given for the form of judgement $a : A$, characteristic of type theory.

It turns out not to be enough to distinguish only the two levels indicated symbolically by A and A true in type theory, calling the latter a judgement, or an assertion, and the former, the content of this judgement, or assertion.

Assertion is a speech act, and speech acts generally have a force/content structure. The content can remain the same while the force changes. A meteorologist might *assert* that the autumn will be mild and dry. A farmer might *fear* this, whereas many of us *hope* that it will be mild and dry. The force can, of course, also remain the same while the content changes. The many assertions making up a scientific paper differ in content, but they all have the same force, namely assertoric force.

A proposition as understood by modern logic is an object of some sort: a truth value, a set of possible worlds, a type of truthmakers, etc. One cannot assert a proposition in this sense: what one can assert is that a proposition is true. Nor can one hope or fear a proposition: what one can hope or fear is that a proposition is true. Likewise for any other speech act: its object, or content, is not a proposition, but rather that a proposition is true. In the force/content analysis of speech acts, it is therefore not the notion of proposition that plays the role of content. Rather, in these examples, the content has the form A true. A speech act, such as assertion, arises by the application of force to such a content.

Martin-Löf (2003) arrived at a three-levelled analysis of the structure of speech acts. If we use the turnstile to symbolize assertoric force, the structure of an assertion, in particular, may be depicted as follows:

$$\underset{\substack{\uparrow \\ \text{assertoric} \\ \text{force}}}{\vdash} \quad \underbrace{\overset{\substack{\text{proposition} \\ \downarrow}}{A}\ \text{true}}_{\text{content}}$$

As Sundholm (2003) notes, this yields three candidates for the operand of a modality: the whole judgement, the content, and the proposition. The most fundamental of these is the first: modality applies first and foremost to the judgement as a whole. We may, however, make sense of its applying both to the content and to the proposition by a process of internalization.

I will concentrate on the modality of necessity, which Kant called the apodeictic modality. This modality can be applied to a judgement provided it has been demonstrated, that is, proved. We are led to regard the following rule as meaning-determining for it:

$$\frac{J}{\overline{\mathrm{Nec}\,J}}$$

The modality Nec applies to judgements. The double line indicates that the premiss judgement, J, must be demonstrated for this rule to be applicable. Clearly, this is just the necessitation rule of standard modal logic raised to the level of judgements.

In a first step of internalization, the necessity modality can be pushed from the level of judgement to the level of content. For content of the form A true, the rule is as follows:

$$\frac{\mathrm{Nec}(\vdash A \text{ true})}{\vdash A \text{ nec-true}}$$

Instead of the truth particle, we here have a particle of necessary truth. The modality in the premiss (apodeictic) differs from the modality in the conclusion (assertoric), hence these are not merely rewritings of each other. The novel form of content, A nec-true, may feature as the antecedent in a hypothetical judgement. This allows us to make sense of assuming a necessity, which we could not do if we had modality only at the judgemental level.

In a second step of internalization, the modality can be pushed from the level of content to the propositional level:

$$\frac{\vdash A \text{ nec-true}}{\vdash \Box A \text{ true}}$$

The novel form of proposition, $\Box A$, allows us to make sense of applying the propositional operators to a necessity as operand and to do modal logic in the standard sense. The notion of modality occurring here is, however, a derived one, viz., derived by two steps of internalization from the judgemental apodeictic modality.

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Abelard and the development of connexive logic

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Following the more detailed exposition in Lenzen (2021), it will be argued that Abelard was the first logician who tried to defend "hardcore connexivism". While "humble connexivism" is willing to admit that the characteristic principles of connexive logic fail to hold in the case of "abnormal" propositions (e.g., impossible antecedents or necessary consequents), "hardcore" connexivism insists that absolutely no proposition p implies, or is implied by, its own negation; similarly, no proposition implies, or is implied by, both of two propositions q and q. In his *Dialectica*, Abelard tried to save these principles against counterexamples, as they had been discovered by contemporary logicians, by requiring that the consequent of a conditional must be "contained" in the antecedent. In particular he denied that a negative property like not being a stone is ever contained in a positive property like being a man. According to Martin (1986), however, Alberic of Paris invented a counterexample to show that, on the basis of logical principles accepted by Abelard, the impossible antecedent 'Socrates is a man and Socrates is not an animal' entails its own negation.

In this talk I will examine three ways out of "Alberic's trap" as they have been attempted by the medieval schools of the *Montanes*, the *Melidunenses*, and the *Porretani*. The first solution (developed in the *Introductiones Montane Minores*) is based on the idea that the conjunction of an affirmative proposition and a negative proposition entails only the former, but not the latter, "because a negation is not so powerful (*vehemens*) when joined with an affirmation as it is when it is alone, and something follows from a negation alone which does not follow from it when it is conjoined with an affirmation" (Martin 2004, p. 198/9). However, the following example which was meant to support this thesis fails to be conclusive. To be sure, "from the negation 'Socrates doesn't dispute' when conjoined with the affirmation 'when Plato is reading' it doesn't follow 'Socrates doesn't discuss with anybody', although this follows when the proposition is put forward *per se*" (de Rijk 1967, p. 66), but this example cannot be transferred to Alberic's considerations.

The second solution (as suggested in the *Ars Meliduna*), consists in the extreme thesis that no consequent at all follows from an impossible antecedent ("ex falso nil sequitur"). In particular, the followers of Robert of Melun believed that the admission of conditionals with impossible antecedents would lead to outright inconsistencies: "If one assumes the opposite, it can be proven that a proposition entails its own negation, that two contradictories follow from one and the same proposition, and that a proposition entails another proposition which cannot be true together with it. Each of these conclusions seems to be against the art, for just as nothing can both be [true] and not be [true], so it cannot be the case that two [contradictories follow from] one and the same proposition" (cf. Iwakuma 1993, p. 142–143). As has been explained at greater length in Lenzen (2022), undoubtedly no proposition q can be both true and false, and therefore q and $\neg q$ cannot be implied by one and the same *true* proposition p. But this does *not* mean that q and $\neg q$ cannot simultaneously be implied by *any proposition at all*. It is not "against the art" that the self-contradictory conjunction $(q \wedge \neg q)$ implies both q and $\neg q$.

The solution of the *Porretani* (i.e., the followers of Gilbert of Poitiers) amounts to a rejection of the laws of conjunctive simplification. In their opinion, whoever holds that if $(p \to q)$, then also $(p \wedge r \to q)$, commits, as they say, the fallacy "non causa ut causa": The conjunctive antecedent $(p \wedge r)$ is “preposed” as the *cause* of the consequent q, but the true cause of q is p alone, while $(p \wedge r)$ is a “*not-cause*”. As Martin (1987, p. 397–398) remarked, the reservations of the *Porretani* concern “exactly the point made by Everett Nelson in his account for the intensional relationship, holding between the antecedent and consequent of a true conditional”. Therefore, in this talk, it will also have to be checked whether Nelson’s approach in (1930) is apt to justify “hardcore” connexivism.

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Semantic pollution of proof systems

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Proof theory and model-theoretic semantics provide different ways of proving results about logics, and soundness and completeness proofs reveal an intrinsic connection between these methods. However, Avron (1996) writes that a requirement of a good proof system is that it should be independent from any particular semantics. This has become known as syntactic purity of a proof system, as opposed to a semantically polluted one. The value of soundness and completeness proofs seems to come from a certain independence that the syntactic side has from the semantic side. If a proof system is semantically polluted, this may take away from its proof-theoretical nature and the expected generality (Avron, 1996). Labeled proof calculi are a standard example of semantically polluted systems, due to their internalization of Kripke semantics into the proof system (see e.g. Poggiolesi & Restall (2015)), but other examples can be found in the literature, including for instance semantic sequents and tableaux (Poggiolesi, 2011), or the inclusion of neighborhood semantics into the proof system (Negri, 2017).

The goal of this talk is to provide a better conceptual characterization of what semantic pollution is, and to provide and compare ways of telling when a proof system is semantically polluted or not. This contributes to a better clarification of what a 'good' proof system can be, and encourages a more nuanced understanding of the distinction between syntax and semantics. The literature distinguishes between a strong and a weak definition of syntactic purity. *Strong* syntactic purity occurs when a proof system is "independent of any particular semantics" (Avron, 1996). This includes the idea that "one should not be able to guess, just from the form of the structures which are used, the intended semantic of a given proof system. *Weak* syntactic purity, on the other hand, says that a sequent calculus cannot make use of 'explicit semantic elements' (Poggiolesi, 2011). Poggiolesi argues that strong syntactic purity is too strong, since it implies that basic propositional sequent calculi already must be declared semantically polluted. Thus, she adopts weak syntactic purity, where she defines a 'semantic element' as an untranslatable ingredient of a sequent. This rules out, for example, expressions like xRy in labeled calculi, that explicitly incorporate the notion of possible worlds and the Kripke accessibility relation.

In this talk, we first consider ways in which an object language can be polluted by additional syntax, generally. After that, we investigate whether we can make sense of semantic pollution as an instance of general pollution. We discuss the different conceptions of semantic pollution, and connect them to possible formal measures. For weak syntactic purity, we discourage the idea that translatability is decisive in the formal description of a semantic element. Instead, we aim to spell out conditions on the (use of) formal language in a proof system in order to exclude semantic elements, which also helps us understand better why these elements are excluded. Inspiring such requirements is the idea that symbols should not refer to something outside of the informal reasoning they formalize, and that the level of 'explicitness' of representation of semantic elements is important. On the latter topic, Poggiolesi & Restall note that elements from Kripke semantics are treated explicitly in labeled systems, but are made implicit in tree-hypersequent systems (reducing the level of semantic pollution) (Poggiolesi & Restall, 2015). Read objects that even in tree-hypersequent systems, "the content is still there" (Read, 2015). We argue that the particular presentation of content in the syntax does indeed matter for semantic pollution. As for strong syntactic purity, we suggest that it might not be *too* strong after all, and we mention some possible formalizations of 'recognizing' semantics from a proof system. For example, the way that proof rules determine the semantics of the logical connectives they define might relate to semantic pollution. In general, each measure can be considered either as a way of going from proof rules to semantic truth conditions, or as a way of incorporating a semantics into proof rules.

We illustrate the ideas above by comparing the measures for various proof systems, including several generalizations of sequents, a neighborhood proof system (see Negri (2017)) and of course labeled systems, and we reflect on whether our measures of semantic pollution seem to interact with any other philosophical properties.

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Generalized ultraproducts for positive logic

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We present a generalization of the ultraproduct construction suitable for positive logic in the sense of Ben Yaakov and Poizat (2007). While the classical notion of ultraproducts involves ultrafilters in the powerset Boolean algebra of some index set, the generalized notion of (prime) *filter products* is dependent on (prime) filters, respectively, in the *Heyting* algebra of up-sets of an index set, which is now equipped with a partial order. In the present abstract, we first define this new notion, and then proceed to present counterparts of classical theorems on ultraproducts, such as Łoś's Theorem, the Łoś-Suszko Theorem, and the Keisler–Shelah Theorem.

Definition.

1. A *wellfounded forest* is a poset in which every principal down-set is a wellorder.

2. Let $(I, \leq)$ be a wellfounded forest, F a filter in $\mathrm{Up}(I, \leq)$, and $(h_{ij} : M_i \to M_j \mid i \leq j \in I)$ be a direct system of homomorphisms between structures of a common signature. Define the set $\prod_F A_i$ to be

   $$\{a \in \prod_{i \in I} M_i : \exists I' \in F \, \forall i \leq j \in I' \, h_{ij}(a(i)) = a(j)\}.$$

   The relation defined by ($a \equiv_F b$ iff $[\![a = b]\!] \in F$) is a congruence of $\prod_F M_i$. We call $\prod_F M_i / \equiv_F$ a *filter product* of $\{M_i : i \in I\}$ and a *prime (filter) product*, when F is a prime filter of $\mathrm{Up}(I, \leq)$.

Remark.

1. Ultraproducts can be regarded as a special kind of prime filter products whose index sets are equipped with the diagonal binary relation.

2. This definition of prime filter products can be motivated in a sheaf-theoretic manner. Sheaf-theoretically, an ultraproduct $\prod_D M_i$ is the stalk at D of a sheaf obtained by composing the topological embedding of I into the Stone space of $\mathscr{P}(I)$ with the sheaf associating each finite subset $I' \subseteq I$ with the direct product $\prod_{i \in I'} M_i$. Naturally generalizing this, one can think of the stalk at D of a sheaf obtained by composing the topological embedding of a poset I into the Priestley space of the $\mathrm{Up}(I)$, where F is a prime filter of $\mathrm{Up}(I)$, with the sheaf

   $$I' \mapsto \{a \in \prod_{i \in I'} M_i \mid \forall i \leq j \in I' \, h_{ij}(a(i)) = a(j)\}.$$

   One can easily check that this stalk is nothing but the prime filter product $\prod_F M_i$.

To state our results, we need to define the following fragment of first-order logic studied in positive logic.

Definition (Ben Yaakov and Poizat (2007))**.**

1. A *positive existential* (or $\exists^+$) formula is a first-order formula obtained by existentially quantifying, finitely many times, disjunctions of conjuctions of atomic formulae. We assume that a first-order language always contains as a 0-ary predicate $\perp$, the contradiction.

2. A basic *h-inductive* formula is a first-order formula obtained by universally quantifying, finitely many times, a conditional between $\exists^+$ formulae. An *h-inductive* (or $\forall_2^+$) formula is a conjunction of basic h-inductive formulae.

Then we have the following analogue of Łoś's Theorem.

Theorem. Given a direct system $(h_{ij} : M_i \to M_j)$ of homomorphisms indexed by a well-founded forest I and a filter F in $\mathrm{Up}(I)$, for every positive primitive formula $\phi(\overline{x})$ (that is, a $\exists^+$ formula with no nontrivial disjunctions) and a tuple $\overline{a}$ of elements of the filter product $\prod_F M_i$,

$$\prod_F M_i \models \phi(\overline{a}) \iff [\![\phi(\overline{a})]\!] \in F.$$

If F is prime, the displayed biconditional is true of all $\exists^+$ formulae.

One can also characterize the definability of a class of structures by $\forall_2^+$ sentences à la Łoś-Suszko.

Theorem. A class K of algebras is axiomatized by $\forall_2^+$ sentences if and only if K is closed under ultraroots and prime products.

We conclude this abstract by stating a counterpart of the Keisler-Shelah Theorem.

Definition. An L-theory T has *the model-wise clique property* if there are a $M \models T$, an infinite $A \subseteq M$, and a positive L-formula $\phi(x,y)$ such that for $a,b \in A$:

$$M \models \phi(a,b) \iff a \neq b.$$

in addition to

$$M \models \neg \exists x\, \phi(x,x)$$

Theorem. Let T be an h-inductive L-theory *without* the model-wise clique property. and $A,B \models T$ be sufficiently saturated. The following are equivalent:

1. $\mathrm{Th}_{\exists^+}(A) = \mathrm{Th}_{\exists^+}(B)$.

2. A and B have some common prime power.

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Paraconsistent and paracomplete Zermelo-Fraenkel set theory

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The classical Zermelo-Fraenkel axioms ZFC are generally accepted as the foundation of mathematics. ZFC is formalized in classical logic in which any statement is either true or false, and cannot be both at the same time. Throughout the course of the 20th century, there has been a continuous interest in foundations of mathematics formalized in various non-classical logics. The most notable examples are constructive set theories, such as IZF, CZF and CST. These theories are based in intuitionistic logic and aim to formalize the constructive side of mathematics.

A more extreme departure from classical logic would be a *paraconsistent* set theory, i.e., a set theory in which statements can be both true and false at the same time. For such a theory to be non-trivial, the underlying logic must, at the very least, fail to satisfy the *ex falso quodlibet* principle. A common motivating factor in this approach is the desire to adopt some form of *full comprehension* as an axiom, and avoid Russells paradox.

In this talk, we are explicitly *not* concerned with full comprehension. In our view, the paradoxes of full comprehension arise from having an implication connective that accurately captures the notion of deductive logical consequence. The prize for full comprehension is therefore too steep. Instead, we prioritise an intuitive treatment of non-classical sets so as to make our theory accessible to the classical mathematician used to working in ZFC.

We propose a natural formalization of set theory in the logic BS4. This is a fourvalued, paraconsistent and paracomplete logic which was first developed in (Dunn, 1976) and (Belnap, 1976, 1977), and elaborated further in (Avron, 1991; Omori & Waragai, 2011; Sano & Omori, 2013). In the semantics of BS4 truth and falsity are formally separated, so a statement φ can be true and not false (1), false and not true (0), both true and false (b), or neither true nor false (n). We formulate an axiomatic system called BZFC, based on a careful generalisation of ZFC, together with the *anti-classicality axiom* postulating the existence of non-classical sets and prove a surprising results stating that the existence of a single non-classical set is sufficient to produce any other type of non-classical set.

In our opinion, previous attempts at a similar approach have not been fully successful. We conjecture that this is, in part, due to an insufficiently careful treatment of the language of set theory. For example, in our logic BS4, there are two types of negations: the *native negation*, which expresses the presence of falsity, and the (defined) *classical negation*, which expresses the absence of truth. Likewise, there are two types of implications: the *native implication*, and the (defined) *strong implication*. When formulating the axioms of our set theory, a careful approach is needed to determine what the proper generalization of each axiom should be.

As a result, we provide a theory with a clear and intuitive *ontology* where a non-classical set u can be described by its *positive-extension* (the collection of all x such that $x \in u$ is true) and *negative-extension* (the collection of all x such that $x \in u$ is false), and this can be expressed within the system. There is, however, an asymmetry between the notion of the positive-extension and the negative-extension. Notice that in ZFC, $\{x : x \in u\}$ is a set but

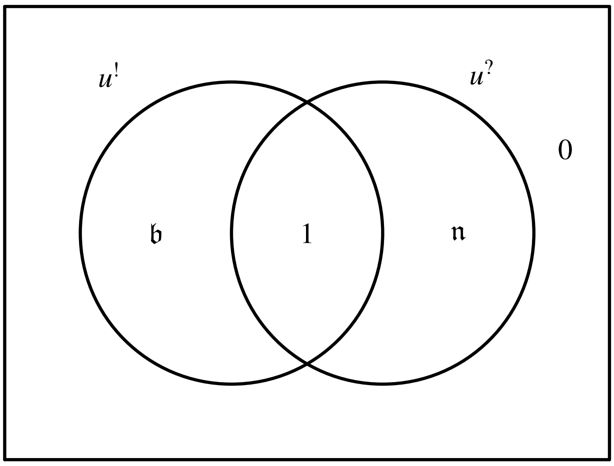

Figure 1: The four truth values of $x \in u$ depending on the boolean combination of $u^!$ and $u^?$.

$\{x : x \notin u\}$ is a proper class. Therefore, it turns out to be more appropriate to talk about the complement of the negative-extension, i.e., the collection of all x for which the statement $x \notin u$ is not true (or, equivalently, the statement $x \in u$ is not false). We denote the positive-extension by $u^!$ and the compliment of the negative-extension by $u^?$. Together $u^!$ and $u^?$ completely describe u (Figure 1).

The universe of non-classical sets naturally extends the classical *von Neumann* universe of sets, and every model of BZFC contains within it a natural model of ZFC given by the hereditarily classical sets. On the other hand, starting in ZFC one can produce a natural model of BZFC, leading to intuitive bi-interpretability between the two theories.

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Approaching modalities by quasi-extensional semantics

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Non-deterministic or quasi-extensional semantics were independently created Ivlev and Rescher [Res62, Ivl88]. Their formal properties has been developed and studied in [AL04, AL05, AZ11]. These semantics are a generalization of the usual matrix approach to many-valued logic. In non-deterministic semantics the interpretation of a connective does not uniquely determine the value of a compound formula. Instead, it assigns the the compound formula a non-empty set of values. Hence, these semantics are not fully extensional which makes them an interesting candidate as a natural semantical explication of some of the non-classical logics.

One line of research of non-deterministic semantics revolves around modal logics. In [Ivl88] Ivlev gave non-deterministic semantics for couple of non-normal modal logics.[1] His idea was to use truth-values as means of representing the modal status of a given proposition. On his account, there are four truth-values: necessary true, contingently true, contingently false, and necessary false.[2]

A similar four-valued approach was developed in [Kea81]. Instead of using the non-deterministic semantics directly, he restricts the set of valuations by means of filtration. The proposed filtration method is called the level valuation. Roughly speaking, at level 0 we start with all valuations. At level 1 we get rid of those, which do not make tautologies of 0-level necessary. We call a valuation a level valuation iff it is a m-level valuation for any natural number m.

Kearns proved that the resulting level valuation semantics are sound and complete with respect to **T, S4, S5**. Those results were further developed in [OS16, CLN19]. First, they axiomatized the non-deterministic semantics before the application of the level valuation technique. Secondly, they generalized this approach to 6 valued semantics. By doing this, they were able to find a level valuation semantics that is complete with respect to some extensions of the logic **KD**. Moreover, these results have been further generalized to 8 valued case. According to this idea, one treats the truth, necessity and possibility of a proposition separately. So, in total they are 8 combinations which they use as truth-values. This is nicely summarized by the following table:

Value	**Status of the sentence**
$\mathtt{T}_\Diamond$	$\Box\varphi, \Diamond\varphi, \varphi$ (necessary, possible and true)
$\mathtt{T}$	$\Box\varphi, \neg\Diamond\varphi, \varphi$ (necessary, not possible and true)
$\mathtt{t}_\Diamond$	$\neg\Box\varphi, \Diamond\varphi, \varphi$ (not necessary, possible and true)
$\mathtt{t}$	$\neg\Box\varphi, \neg\Diamond\varphi, \varphi$ (not necessary, not possible and true)
$\mathtt{f}_\Diamond$	$\neg\Box\varphi, \Diamond\varphi, \neg\varphi$ (not necessary, possible and false)
$\mathtt{F}$	$\Box\varphi, \neg\Diamond\varphi, \neg\varphi$ (necessary, not possible and false)
$\mathtt{F}_\Diamond$	$\Box\varphi, \Diamond\varphi, \neg\varphi$ (necessary, possible and false)
$\mathtt{f}$	$\neg\Box\varphi, \neg\Diamond\varphi, \neg\varphi$ (not necessary, not possible and false)

Moreover, in [CLN19] non-deterministic semantics for **KD, KDB, KD4, KD45** have been presented.

[1]In this context by non-normal we mean that these are not closed under the rule of neccessitation.
[2]He also develops two and three-valued modal semantics in this paper.

In our paper we build on these results. First, we show how one can make the non-deterministic semantics modular with respect to the rule of necessiation [NEC]. By this we mean that the logic that is based on the non-deterministic semantics without level valuation is a NEC-free fragment of the final logic. Secondly, we modify the non-deterministic semantics so that the resulting semantics are complete with respect to the logic **K** and any extension of it by any combination of axioms 4,T,B,D,5. Next, we also show that this semantics is flexible enough to invalidate the axiom K. The resulting system which we call **H** is a very weak modal logics that is not even closed under substitution of provably equivalent formulas within the scope of the modal operators. Notably, **H** does not have a finitely-many valued matrix semantics and it is not clear whether one could find a natural possible semantics for it. We prove the completeness theorems with respect to those systems.

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On the computational content of a generalized Harrop rule

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The Harrop rule (Harrop, 1960) also known as the Independence of Premise rule or the Kreisel-Putnam rule:

$$\frac{\neg C \to (A \lor B)}{(\neg C \to A) \lor (\neg C \to B)}$$

is an intriguing rule. It is an admissible but not a derivable rule of intuitionistic logic (Iemhoff, 2001), despite being proof-theoretically valid (Piecha et al., 2014) in the Prawitz-style semantics. If we add it to the intuitionistic logic, we obtain the Kreisel-Putnam logic, which is stronger than the intuitionistic logic yet still has the disjunction property (whenever $A \lor B$ is a theorem, either A or B is a theorem), previously thought to be a property specific to the intuitionistic logic. Furthermore, it is admissible in any intermediate logic (Prucnal, 1979).

Yet, its generalized version, which we call the Split rule (where C is a Harrop formula, i.e., a formula in which every disjunction occurs only within the antecedents of implications):

$$\frac{C \to (A \lor B)}{(C \to A) \lor (C \to B)}$$

is arguably even more interesting. If we add it to the intuitionistic logic, we obtain inquisitive logic (Punčochář, 2016, Ciardelli et al., 2020), which has both the disjunctive property and the structural completeness property (enjoyed by classical logic: every admissible rule is derivable), it can be justified from the perspective of proof-theoretic validity (Stafford, 2021), yet it is not closed under uniform substitution. Furthermore, it is admissible in any intermediate logic (Minari and Wronski, 1988).

Despite its significance, the Split rule itself remains mostly unexplored, especially in terms of its proof-theoretic meaning and computational content (a recent exception to this is (Condoluci and Manighetti, 2018) examining the admissibility of the Harrop rule from the computational view). In this talk, we fill explore this gap and propose a computational interpretation of the Split rule in the style of BHK semantics. We will achieve this by exploiting the Curry-Howard correspondence between formulas and types. First, we inspect the inferential behaviour of the Split rule in the setting of natural deduction system for the propositional intuitionistic logic. This will then guide our process of formulating an appropriate program that would capture the corresponding computational content of the typed Split rule. In other words, we want to find an appropriate selector function (i.e., a noncanonical eliminatory operator) for the Split rule by considering its typed variant. Our investigation can be thus also reframed as an effort to answer the following questions: is the Split rule constructively valid in the style of BHK semantics? In other words, can we find a constructive function that would transform arbitrary proofs of the premise of the Split rule into proofs of its conclusion?

We propose two possible selectors (with appropriate computation rules) corresponding to the two possible generalizations of the typed Split rule: one generalization (the S rule) is based on the selector for the typed disjunction elimination rule, the other (the FS rule) is based on the selector for the typed general implication elimination rule. Both variants are equivalent, but the latter requires the adoption of rules with higher-level assumptions, i.e., assumptions that depend on another assumptions.

The (typed) rule S takes the following form:

$$\dfrac{\begin{array}{ccc} [z:C] & [x:C\to A] & [y:C\to B] \\ c(z):A\vee B & d(x):D & e(y):D \end{array}}{\mathsf{S}(z.c,x.d,y.e):D}\,\mathrm{S}$$

with the computation rules $\mathsf{S}(z.\mathsf{i}(a(z)),x.d,y.e) = d(\lambda z.a(z)/x) : D$ and $\mathsf{S}(z.\mathsf{j}(b(z)),x.d,y.e) = e(\lambda z.b(z)/y) : D$. The rules FS takes the following form:

$$\dfrac{f:C\to(A\vee B) \qquad \begin{array}{c}\begin{bmatrix}[x:C]\\ y(x):A\end{bmatrix}\\ d(y):D\end{array} \qquad \begin{array}{c}\begin{bmatrix}[x:C]\\ w(x):B\end{bmatrix}\\ e(w):D\end{array}}{\mathsf{FS}(f,y.d,w.e):D}\,\mathrm{FS}$$

with the computation rules $\mathsf{FS}(\lambda(\mathsf{i}(a)),y.d,w.e) = d(a) : D$ and $\mathsf{FS}(\lambda(\mathsf{j}(b)),y.d,w.e) = e(b) : D$.

So, can we find a selector for the Split rule? Our answer is positive for its generalized versions S and FS but negative for the Split rule itself. The computational content is expressed by the program S, or, if we allow higher-level assumptions (corresponding to function variables), by the higher-level program FS.

Note that the FS rule has in comparison with the S rule a number of advantages: we do not have to reduce the original premise of the Split into a hypothetical derivation, we can just keep it as it is and treat the rule as an elimination-like rule for implication (in other words, the major premises of the Split rule and the FS rule are the same, which is not the case for the Split rule and the S rule). Furthermore, we do not need to introduce the auxiliary implication assumptions as in the S rule and instead handle the dependency between $A\vee B$ and C more directly via the notion of a higher-level assumption.

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Non-logical meaning and meaning of rules: A symmetry between local and logical validity in Prawitz's semantics

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Prawitz's (1973) semantics of valid arguments (SVA) is a well-known example of proof-theoretic semantics in the field of logical constructivism. SVA can be conceived of as an extension of Prawitz's (1965) own normalisation results for Gentzen's natural deduction systems based on what Schroeder-Heister (2006) called the "fundamental corollary" of Prawitz's normalisation: in certain (important) systems, derivations of A normalise to derivations of A ending by an introduction. This seemingly respects Gentzen's (1935) claim that introductions fix the meaning of the logical constants, whereas eliminations are unique functions of the introductions. Following Dummett's (1993) principle that, if A is provable, then A must also be canonically provable (where "canonical" means "ending by an introduction"), one can thus simply turn the corollary into a semantic requirement, and replace derivations with arbitrary argument-structures $\mathscr{D}$, whose non-introductory inferences be justified by equally arbitrary justification procedures $\mathfrak{J}$. The general idea of SVA is framed by the case where $\mathscr{D}$ is closed, i.e. has no free variables or assumptions: $\langle \mathscr{D}, \mathfrak{J} \rangle$ is valid iff, by applying in some order the elements of $\mathfrak{J}$, $\mathscr{D}$ reduces to a canonical form whose immediate sub-structures are valid when paired with $\mathfrak{J}$.

However, this general idea must be refined through some basic, but crucial intuitions. Firstly, arguments may be simply *locally* valid, e.g. because of some specific meaning of the non-logical terminology they involve; if not, the argument can be said to be *logically* valid. Hence, we have to specify how non-logical meaning is determined, but in doing this, we must comply with a second fact, namely, that argument-structures may be open, so that validity should be defined for the open case too.

In SVA, determination of non-logical meaning is achieved through *atomic systems* Σ, i.e. the meaning of the non-logical terminology is given in terms of deductive use of this terminology in purely atomic derivations. With this established, local validity becomes validity over an atomic system, and validity in the open-case is dealt with through a kind of closure principle, i.e. open arguments are valid when all their closed instances are so. At this point, though, SVA faces with a first dilemma: when closing open arguments, should we require validity over one and the same system, or should we require that the property is preserved over extensions of the system? In other words, we have the following alternatives.

Definition 1. $\langle \mathscr{D}[x_1,..,x_n,A_1,...,A_m], \mathfrak{J} \rangle$ is NE-valid over Σ iff, for every k_i in the language of Σ, for every closed $\langle \mathscr{D}_j, \mathfrak{J}_j \rangle$ valid over Σ, $\langle \mathscr{D}[k_1,...,k_n,\mathscr{D}_1,...,\mathscr{D}_m], \mathfrak{J} \cup \mathfrak{J}_1 \cup ... \cup \mathfrak{J}_m \rangle$ is valid over Σ.

Definition 2. $\langle \mathscr{D}[x_1,..,x_n,A_1,...,A_m], \mathfrak{J} \rangle$ is WE-valid over Σ iff, for every Σ^+, for every k_i in the language of Σ^+, for every closed $\langle \mathscr{D}_j, \mathfrak{J}_j \rangle$ valid over Σ^+, $\langle \mathscr{D}[k_1,...,k_n,\mathscr{D}_1,...,\mathscr{D}_m], \mathfrak{J} \cup \mathfrak{J}_1 \cup ... \cup \mathfrak{J}_m \rangle$ is valid over Σ^+.

This distinction is anything but trivial for, as shown by Schroeder-Heister (2006), it determines whether the overall semantics is or not monotonic.

Proposition 3. $\mathtt{NE}$-validity is non-monotonic, i.e. there is $\langle \mathscr{D}, \mathfrak{J} \rangle$ such that, for some Σ, for some Σ^+, $\langle \mathscr{D}, \mathfrak{J} \rangle$ is $\mathtt{NE}$-valid over Σ and $\langle \mathscr{D}, \mathfrak{J} \rangle$ is not $\mathtt{NE}$-valid over Σ^+.

Proposition 4. $\mathtt{WE}$-validity is monotonic, i.e. for every $\langle \mathscr{D}, \mathfrak{J} \rangle$, for every Σ, if $\langle \mathscr{D}, \mathfrak{J} \rangle$ is $\mathtt{WE}$-valid over Σ then, for every Σ^+, $\langle \mathscr{D}, \mathfrak{J} \rangle$ is $\mathtt{WE}$-valid over Σ^+.

Also, $\mathtt{WE}$-validity permits to equate logical validity with validity over the empty system, which seems in turn to capture in a straightforward way the idea that an argument is logically valid when its validity is independent of the meaning of the non-logical terminology (but see below).

Proposition 5. $\langle \mathscr{D}, \mathfrak{J} \rangle$ is logically valid iff $\langle \mathscr{D}, \mathfrak{J} \rangle$ is $\mathtt{WE}$-valid over $\emptyset$.

Concerning logical validity, however, we have a second dilemma, for there seem to be at least two SVA-compatible ways for understanding independence from non-logical meaning. This stems from the fact that arguments are *pairs* consisting of an argument-structure *plus* a set of justification procedures. Quantification on non-logical meanings must be hence accompanied by quantification on sets of justification procedures, and this returns again two alternatives.

Definition 6. $\Gamma \models_1 A$ iff there is $\mathscr{D}$ with assumptions Γ and conclusion A such that, for every Σ, for some $\mathfrak{J}$, $\langle \mathscr{D}, \mathfrak{J} \rangle$ is valid over Σ.

Definition 7. $\Gamma \models_2 A$ iff there is $\mathscr{D}$ with assumptions Γ and conclusion A such that, for some $\mathfrak{J}$, for every Σ, $\langle \mathscr{D}, \mathfrak{J} \rangle$ is valid over Σ.

My aim in this talk is twofold. First, I argue that the options in the alternatives above have advantages and shortcomings which are symmetric both from an internal, and from an external point of view – i.e. both within the same alternative, and with respect to the other one.

As we have seen, $\mathtt{NE}$ has the shortcoming of returning a non-monotonic notion of validity, which seems to be in contrast with the idea that if an argument is valid, it should remain so when expanding the knowledge base. But $\mathtt{NE}$ has also the advantage of accounting for the idea that arguments may be valid, not only thanks to their inferences, but also because of the meaning of their non-logical vocabulary. $\mathtt{WE}$ has the advantage of ensuring monotonicity but, since atomic systems fix non-logical meaning, expanding atomic systems in the open case implies changing this meaning, so $\mathtt{WE}$ has also the shortcoming of relaxing too much the idea that validity depends on given non-logical features. This is seen from proposition 5: that logical validity amounts to validity over the empty system means that an argument is logically valid, not when it is so *whatever* non-logical signs mean, but when this meaning *plays no role* in deduction.

This sort of priority of the "logico-deductive" aspects of validity is also a shortcoming of $\models_2$. $\Gamma \models_2 A$ means that we have an argument-structure from Γ to A whose non-introductory inferences are justified in *one and the same* way over all systems, regardless of how justification procedures interact with atomic rules. Our argument is logically valid, not in the sense of being justifiable for every determination of non-logical meaning, but in the sense of remaining justified throughout variations of this meaning. The advantage of $\models_2$ is of course that this seems to be what we have in mind when we speak of a "universally" valid argument, i.e. an argument whose justification needs not be adapted to non-logical constraints. This stability of validity is similar to the monotonic character of local validity, granted by $\mathtt{WE}$. In contrast, $\models_1$ looks at logical validity in a much more "model-theoretic" sense, i.e. as justifiability relative to determinations of non-logical meanings. This is a natural generalisation of local $\mathtt{NE}$-validity, and accordingly one where “universal” validity is not correctness of justified inferences irrespective of non-logical meanings, but adaptability of these inferences to those meanings.

If one accepts this reconstruction, one may be also tempted to conclude that, if we adopt NE (resp. WE) at the local level, we should then adopt $\models_1$ (resp. $\models_2$) at the “global” level. However, as a second aim of my talk, I suggest that the aforementioned symmetries stem from a deeper duality in Prawitz’s semantics, i.e. meaning of non-logical terminology *vs* meaning of rules, where the latter is interpretation of inferential transitions through suitable justification procedures. In particular, meaning of rules can either *interact* with non-logical meaning, or *be tested over* a semantic class of potential non-logical meanings. Based on this more basic opposition, the mixed readings NE $+ \models_2$ and WE $+ \models_1$ become acceptable - and indeed, are actually found in the literature.

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Notes on the proof theory for relevant logics: From Routley-Meyer semantics to labelled sequent systems

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Introduction & aim Relevant logics are a well-known family of non-classical logics introduced to cope with so-called paradoxes of material implication. According to relevantists, $\rightarrow$ is intended to express a more fine-grained and philosophically motivated notion of conditional than material implication. Part of the philosophical intuition of relevant logics, at least in the early development by Anderson,Belnap (1965), was that the antecedent and consequent of a valid conditional must be relevant to each other, in the sense that, in expressions of the form $A \rightarrow B$, there must be a strong connection between antecedent and consequent.
Relevant logics attracted much attention over the years – especially, from the perspective of axiomatic systems and semantic structures (see e.g., Dunn,Restall (2002)). In this talk, we aim at enriching the set of perspectives under which relevant logics can be considered. Therefore, we will consider a proof-theoretic characterization of relevant logics by employing the methodology of labelled sequents (see, e.g., Negri (2005),Negri (2007)). Labelled sequent systems are rule-based calculi that employ labelled formulas $w : A$ (meaning, "A is forced at w"), and have the accessibility relation (e.g., wRv) explicitly internalized within sequents, by being capable, at the same time, of preserving several desirable structural properties common to sequent systems. More precisely, we will introduce a labelled calculus for the basic relevant logic **B** and, then, explain how to enrich the framework in order to capture relevant logics stronger than **B**.

Routley-Meyer semantics In order to present our intended sequent system, we will introduce so-called *Routley-Meyer models* for relevant logics (see, e.g., Routley et al. (1982); Slaney (1987); Priest (2008)). Such structures are generalizations of Kripke models for modal and intuitionistic logics. In Routley and Meyer's approach, however, we will consider ternary relations instead of binary ones, introduce an additional operation to cope with negation and a distinguished world, denoted 0. Formally, $\mathscr{M} = \langle W, R, ^*, 0, v\rangle$ is a Routley-Meyer model, where W is a set of points, with 0 as a distinguished element, R is a ternary relation on W and * is the *star* operator, such that $^* : W \mapsto W$. Finally, by letting $\Vdash$ be the forcing relation we can provide semantic clauses for $\sim$ and $\rightarrow$ as follows:

$$\mathscr{M}, a \Vdash {\sim}A \quad \text{iff} \quad \mathscr{M}, a^* \nVdash A \tag{$\sim$}$$

$$\mathscr{M}, a \Vdash A \rightarrow B \quad \text{iff} \quad \forall b, c \in W, \text{ if } Rabc \text{ and } \mathscr{M}, b \Vdash A, \text{ then } \mathscr{M}, c \Vdash B \tag{$\rightarrow$}$$

Furthermore, each relevant logic is characterized by some constraints imposed on R. For our purposes, we will consider Routley-Meyer models for basic relevant logic **B** with the following conditions on R:

$$R0aa \quad \text{(F1)}$$

$$R0ab \wedge R0bc \to R0ac \quad \text{(F2)}$$

$$R0da \wedge Rabc \to Rdbc \quad \text{(F3)}$$

$$a^{**} = a \quad \text{(F4)}$$

$$R0ab \to R0b^*a^* \quad \text{(F5)}$$

All these considerations will lead us to the main section of the talk.

Labelled sequent system Our aim is to formulate rules for relevant logics in terms of labelled sequents whose peculiarity is, as said, that of internalizing semantic objects within the syntax.

Exactly by starting from the semantic clauses and the constraints on R sketched above, we will introduce **GrB** ("**G**entzen system for **r**elevant logic **B**"), i.e., a labelled sequent system with height-preserving invertible right and left rules for basic relevant logic **B**. More precisely, the rules for negation $\sim$ and relevant implication $\to$ will have the following shape:[1]

$$\frac{\Gamma \Rightarrow \Delta, a^* : A}{a : \sim A, \Gamma \Rightarrow \Delta}\ L\sim \qquad \frac{a^* : A, \Gamma \Rightarrow \Delta}{\Gamma \Rightarrow \Delta, a : \sim A}\ R\sim$$

$$\frac{Rabc, a : A \to B, \Gamma \Rightarrow \Delta, b : A \qquad Rabc, c : B, a : A \to B, \Gamma \Rightarrow \Delta}{Rabc, a : A \to B, \Gamma \Rightarrow \Delta}\ L\to$$

$$(b, c \text{ fresh})\ \frac{Rabc, b : A, \Gamma \Rightarrow \Delta, c : B}{\Gamma \Rightarrow \Delta, a : A \to B}\ R\to$$

Finally, by starting from the conditions on R (i.e., F1–F5), we add the following mathematical rules to obtain **GrB**:

$$\frac{R0aa^{**}, R0a^{**}a, \Gamma \Rightarrow \Delta}{\Gamma \Rightarrow \Delta}\ \textsc{Inv} \qquad \frac{R0aa, \Gamma \Rightarrow \Delta}{\Gamma \Rightarrow \Delta}\ \textsc{Ref} \qquad \frac{R0b^*a^*, R0ab, \Gamma \Rightarrow \Delta}{R0ab, \Gamma \Rightarrow \Delta}\ \textsc{Cont}_0$$

$$\frac{R0ac, R0ab, R0bc, \Gamma \Rightarrow \Delta}{R0ab, R0bc, \Gamma \Rightarrow \Delta}\ \textsc{Tran}_1 \qquad \frac{Rdbc, R0da, Rabc, \Gamma \Rightarrow \Delta}{R0da, Rabc, \Gamma \Rightarrow \Delta}\ \textsc{Tran}_2$$

Our goal is to give a comprehensive analysis of the structural properties of **GrB** (height-preserving admissibility of the structural rules and height-preserving invertibility of the logical rules), as well as a comparison with other related works (e.g., Viganò (2000); Kurokawa,Negri (2020)). We will conclude the section by showing the first main result of the talk, namely that *Cut* is an admissible rule of **GrB**.

Modularity & conclusion In conclusion, we will consider some methodological questions connected to labelled systems and suggest how the framework proposed throughout the talk can be accommodated to characterise stronger relevant logics by giving some concrete examples.

[1]The calculus includes also axioms of the form $R0ab, a : p, \Gamma \Rightarrow \Delta, b : p$ (for p atomic) and labelled rules for $\vee$ and $\wedge$.

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Theory of predicables and sortal logic

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The aim of the paper is to investigate the scholastic theory of predicables in a system of modern logic, namely, sortal logic (SL). Theory of predicables (TP) is a classification of a certain type of predicates (first-order, monadic, simple). TP mainly establishes five different terms, genus, species, difference, property and accident. Furthermore, a traditional distinction is made between essential and nonessential predicables. Essential predicables include genus and species, and nonessential ones include accident, property, and difference. TP is a traditional part of logic, it is closely linked to philosophy and used in philosophy, and moreover, the distinction between different types of predicates seems to be supported by natural language and argumentation (terms like species, property etc. are common even in everyday usage, apart from Linn classification). It is also very common to treat concepts as subordinated and superordinated. The difference between essential and non-essential predication is also of great philosophical importance. Attempts to revive and reconstruct this doctrine could be useful and beneficial for the current natural and philosophical language analysis, to say nothing of the history of logic, in which is TP of an immense historical importance.

First-order logical systems (classical or otherwise) usually do not distinguish within this group of predicates. In SL, a distinction is made between sortal and non-sortal predicates and accordingly between sortal and standard predication. Sortal predication is then simply a predication with sortal predicates and standard predication is a predication with non-sortal predicates.

Genus is traditionally that under which a species is ordered, species is what is ordered under a genus. Here comes the basic thesis of interpretation in SL: let the species in TP be understood as sortals. To have a proper understanding of the concept of species, it is necessary to add that the concepts of species and genus in TP are correlative. E.g., man is a species of animal and at the same time, the animal is a species of body. This is true in TP for every species and genus, except for the so-called lowest species, i.e., species that no longer divide themselves into other species and the highest genera, i.e. genera that are no longer species of some superior genus. Central thesis is therefore formulated for species, but it means precisely the correlative concept of species, which includes all genera and species except highest genera. Now, species can be characterised as a compound $\Phi(\alpha,\beta)$, where Φ consists of two concepts, superordinate sortal (genus proximum) α and specific difference (differentia specifica) β. α is a sortal, so connected with a principle of identity, i.e. according to Strawson (1959) “a principle for distinguishing and counting individual particulars”. β is – in Geach (1962) terminology – an adjectival term. As Dummett (1973) suggests, sortal is also connected with a criterion of application, that which determines when it is correct to apply a predicate to an individual. Most adjectives, according to Dummett, are connected only with a criterion of application. So β shares the same criterion of application as “its” α. So, specific differences are always genus-relative, i.e. are meaningfully applicated only to the members of the given superordinated sortal (= genus).

A semantic model for the system is a structure $\langle D,E,S,A\rangle$, where D is a non-empty set, $S \subseteq \mathscr{P}(E)-\{E\}$) (where “$\mathscr{P}(E)$” stands for the power set of E), A is an assignment function from the set of individual constants and sortal term constants into $D \cup S$, such that $A(t) \in D$, if t is an individual constant and $A(L) \in S$, if L is a sortal constant. We can understand the predication of genus and species as cases of essential predication, of the form: Every man is animal – $(\forall yD)(\exists xA)(x =_S y)$.

Interesting result is that traditionally in SL negation of a sortal is not a sortal. If by negation of a sortal we mean external (Boolean) negation of a sortal (negation of the whole Φ), then the result is not a sortal. Internal negation of a sortal S, say $\sim S$, is a sortal which consists in a set of properties Φ'. Φ' includes the same α as S, but contains negation of β.

Difference, property and accident can be jointly taken as non-sortal predicates. Disntinction is (as in the case of essential predicates) in their role in (standard) predication. Generally, standard predication will be taken as follows: Every dog is white – $(\forall yD)Wy$.

Difference (specific) is traditionally that in virtue of which we divide genera into species. Specific differences are always genus-relative, i.e., are meaningfully applicated only to the members of the given superordinated sortal (= genus). Now, when we predicate a difference about some object, then we claim a) that it falls under the sortal (and thus determine the criteria of identity a) and b) that we determine the object thus identified in more detail using some non-sortal predication. a) a is a case of sortal predication, b) is a case of standard predication.

A property is traditionally a predicate which does not indicate the essence of a thing, but yet belongs to that thing alone, and is predicated convertibly of it; accidents are items which come and go without the destruction of their subjects. Property and accident are also non-sortal predicates, i.e., are always genus-relative. A special feature of properties is their coextensionality with the species, which is the cause of the convertibility mentioned in the definition. At the intensional level, it is thus possible to understand the properties as following from the concept of the respective species, as has traditionally been the case with the ability to laugh, which is a consequence of human rationality.

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The smallest modal system and its extensions

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Over the last decades logicians like Krister Segerberg (1971), David Makinson (1973) Heinrich Wansing (1989), Lloyd Humberstone (2016) started their investigations of modal logics with a very a weak system of modal logic, they either call $\mathbf{L}_0$, **PC** or **S**.[1] Following Humberstone in (Humberstone, 2016, p. 18) we define the smallest *modal* system as a set **M** in the language of classical propositional logic (**CPL**) containing one additional unary operator, usually $\Box$, where $\Diamond$ is defined as usual, satisfying the following properties:

- **M** contains all (classical) tautologies
- **M** is closed under uniform substitution
- **M** is closed under Modus Ponens

It is obvious from this definition that no specific reference to the primitive modal operator $\Box$ is present, which is usually justified in the sense that one was looking at the *broadest possible definition of what a modal logic might be*, cf. Humberstone (2016). The joint idea of how to proceed with the smallest modal system would then be to extend the axiomatization by certain rules or axioms for the modal operator and thus gaining a structured way of describing various systems of modal logic.

What is missing in all those representation for the smallest modal system is a semantics that differs from the semantics of **CPL**, by giving some meaning to the modal operator, which is kept for all extensions of the smallest modal system, because usually the shift to other (well-known) semantics like Kripke semantics or neighborhood frames is made rather abruptly. However, in what follows, we will provide such a semantics, by building upon the framework of non-deterministic semantics, which was systematically introduced by Arnon Avron and his collaborators, cf. Avron and Zamansky (2011), but already used in the context of modal logics by Yuri Ivlev and John Kearns, cf. Ivlev (1988, 1991), Kearns (1981, 1989), and further developed more recently for example in Coniglio et al. (2020), Grätz (2022), Omori and Skurt (2016, 2021), Pawlowski and La Rosa (2022).

We assume a propositional language $\mathscr{L}$, consisting of a finite set $\{\neg, \rightarrow, \circ\}$[2] of propositional connectives and a countable set of propositional parameters. Furthermore, we denote by Form the set of formulas defined as usual in $\mathscr{L}$. We denote formulas of $\mathscr{L}$ by A, B, C, etc. and sets of formulas of $\mathscr{L}$ by Γ, Δ, Σ, etc.

Definition 1. An **M**-relational-interpretation is then a relation ρ, between formulas and the values 1 and i (i.e., $\rho \subseteq \mathsf{Form} \times \{1, i\}$) such that ρ satisfies the following:

$$\begin{array}{rcl} \neg A\rho 1 & \text{iff} & \text{not } A\rho 1 \\ A \rightarrow B\rho 1 & \text{iff} & \text{not } A\rho 1 \text{ or } B\rho 1 \\ \circ A\rho 1 & \text{iff} & A\rho i \end{array}$$

[1] In the more recent paper by Grätz (2022) this system is called **0** with the intended reading that there are zero restrictions for the modal operator.

[2] We prefer to use $\circ$, rather than $\Box$ or $\Diamond$ because $\circ$ does not have any properties yet and can be interpreted as either.

Then, A is a *relational* $\mathbf{M}$-*consequence* of Γ ($\Gamma \models_{\mathbf{M}} A$) iff for every $\mathbf{M}$-relational-interpretation ρ, if $B\rho 1$ for every $B \in \Gamma$ then $A\rho 1$.

The first two conditions on ρ reflect the (classical) truth-conditions for the non-modal operators $\neg$ and $\rightarrow$, while the third condition says that any modalized formula A is true, i.e., related to 1, iff A is related to the modal value i. This semantics is not unlike the one for Belnap-Dunn logic, more commonly known as **FDE**, cf. Dunn (1976), with the main difference, that nothing is said about formulas standing in relation to i, that is where the non-determinacy comes into play. However, we do not want to treat i as a semantic value that is supporting somewhat truth or falsity. A formula related to i has some modal flavor, what this is supposed to be is at the level of the smallest modal system $\mathbf{M}$ not determined, yet. The properties of $\circ$ will be added with the addition of modal axioms and rules.

Based on this the aim of this paper is then as follows, we will show how to develop in a structured way various sound and complete systems of normal and non-normal modal logics, including systems of modality for which no Kripke semantics is available. What we then hope to achieve is a non-standard approach to modal logics which is in some cases more expressive than the standard treatments of modality in terms of possible worlds.

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Logical pluralism and genuine logic

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Beall & Restall (2020) (hereafter B&R) suggested a version of logical pluralism that does not consist in pointing out the trivial fact that logicians study many different logical systems. They contrast their pluralism with the Carnapian one, which is a matter of the principle of tolerance: "In logic, there are no morals. Everyone is at liberty to build his own logic, i.e. his own form of language, as he wishes." B&R try to convince us that there are several legitimate (and possibly competing) versions of the relation of logical consequence, all of which (a) are equally correct and (b) are hosted by the same language.

According to B&R, an argument is correct iff it preserves truth in every case; viz. if there is no case when its premises are true and the conclusion false. The concept of a case thus plays a central role. What are the relevant cases? B&R argue that there is no single correct answer to this question, it can be answered in more than one way, and consequently we have more than one theory that deserves to be accepted as a logic. They propose what they call the Generalized Tarski Thesis (GTT): An argument is valid$_x$ if and only if, in every case$_x$ in which the premises are true, so is the conclusion. Classical logic (CL) is yielded by taking the cases to be Tarskian models or possible worlds. Intuitionistic logic (IL) is yielded by taking the cases to be constructions. Relevant logic (RL) is yielded by taking the cases to be situations.

Thus, e.g.,

(DN) $\dfrac{\neg\neg A}{A}$

is valid in CL and RL, but invalid in IL; while

(DS) $\dfrac{\neg A \quad A \vee B}{B}$

is valid in CL and IL, but invalid in RL. The argumentation of B&R thus warrants a specific version of logical pluralism as we can (and perhaps must) choose from (at least) three different logics.

We argue that to properly evaluate the argumentation of B&R (as well as many of their critics), we must understand which language (or languages) are in our focus - in particular in which language we find the arguments the forms of which are spelled out by (DN) and (DS). If it is an artificial language formed by logicians (like a language of classical propositional logic), then the validity of (DN) or (DS) is a stipulative matter (and consequently the debate whether they really do hold is either trivial or pointless). For such a language, also, the concepts of truth and truth-preservation are illusory: the only way a sentence of such a language may become "true" is by being designated as such by fiat, resp. by decision of the creator of the system.

If the language in question is meant to be a natural language – or an artificial language firmly tied to natural language as its "regimentation" – then it should be clear for which expressions of the natural language the crucial symbols like "¬" and "∨" (etc.) stand. Then, however, (DN), (DS) and the like should hold for prototypes of "¬" and "∨" in the given natural language - for it is sameness (or near sameness) of inferential roles of the artificial constants with those of their natural language counterparts that lets the former regiment the latter. And whether this is the case, as a natural language is an empirical phenomenon is not a question that logicians could answer, it is rather a question for empirical linguists (and we shouldn't be surprised if their study of the language won't yield definite answers).

Is there another possibility? Can an expression like "¬" or "∨" be equipped with a certain meaning in another way than a) by a set of deliberate conventions produced by a creator of the relevant language, b) by establishing that they stand for certain expressions of natural language as their shortcuts? There may seem to be another quite natural option – we can take the expressions stand as representations or perhaps approximations of a kind of "genuine" negation, "genuine" disjunction etc. True, we maybe do not know what exactly a genuine negation is, and whether there is only one or more, but is not the business of logic to find out precisely this?

We argue that it is not, for there is no way to find this out. We argue that there is nothing like "genuine" logical constants, "genuine" logical consequence, "genuine" logic, or "genuine" logical language. There are only natural languages, which we use to argue and reason, and the artificial languages which can serve us - better or worse - as their regimentations. What B&R say suggests that they are after identifying legitimate (and in some sense privileged) variants of some primary logical operators – the operators belonging to the (alleged) language which is common for CPL, IL and RL.

We want to argue that their project is misconceived as it is based on the idea that we can put aside the Carnapian pluralism and identify a true pluralism which is more authentic. This is, in our view, a seductive but potentially misleading illusion. In our view, we can form multitude of artificial languages which are meant to help us to overcome the ambiguity and indistinctness of natural languages (and there is, needless to say, irreducibly many natural languages). There is no one of them that we could pick up and say: "This is the one where the serious business of logic should be done. Let us find out whether it harbors only one logic or more." As logic is always tied to a particular language, we cannot say how many (genuine?) logics there are, for we would have to say how many (genuine?) languages of logic there are, and this is something we cannot do.

What is closest to a "genuine" logic, for a person, is the logic implicit to the natural language they use to argue and reason. But this is a mere "protologic", which is a) is not articulate enough to act itself as logic in the standard sense, and b) slightly varies among different natural languages and so persons with different mother tongues may have (slightly) diverse logical intuitions. Therefore, we must create our artificial languages as means of its commonly acceptable regimentations in a process of zooming in on a reflective equilibrium: of turning the "protologic" into a logic proper. But there are various ways of doing this, so here there is lot of a space for the Carnapian pluralism but not for a kind of "more genuine" pluralism.

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The algebraic structure of Mares-Goldblatt models

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This talk concerns the Mares-Goldblatt semantics for first-order logics — this semantics, defined in (Mares & Goldblatt, 2006) gets around the fact, noted by Fine (Fine, 1989), that the standard first-order versions of strong relevant logics, like those between **T** and **R**, are incomplete with respect to the constant-domain semantics over their ternary relation frames. It has been developed for a range of relevant logics (Ferenz, forthcoming; Tedder & Ferenz, 2022), but to some modal logics as well (Goldblatt & Mares, 2006; Goldblatt, 2011). This semantics is somewhat more complex than the usual constant domain semantics, or even variable domain semantics, commonly used in modal logic, but it provides for a much more general toolkit for capturing first-order extensions of logics characterised by classes of frames. It employs a *general frame* (or *admissible proposition*) construction, along with a style of evaluation borrowing from Halmos' (Halmos, 1962) *polyadic algebras*. Of particular importance is that we do not interpret quantifiers in terms of the generalized intersections/unions of their instances, but rather we introduce new operations which mimic the key behaviours of these, while allowing us to avoid the problems with constant domains.

In this talk, I'll characterise the underlying *algebraic* structure of Mares-Goldblatt frame, and models, by focusing on how the general frame machinery is employed to allow us to talk about quantifiers by appeal to two different complex algebras on one and the same frame. The basic idea is to note that in a Mares-Goldblatt frame, we pick out a complex algebra of a frame (not necessarily the *full* complex algebra), and we evaluate quantifiers within that structure by appeal to the full complex algebra, in which the substructure embeds. A natural way to capture this interaction algebraically speaking is as follows:

Definition 1. *An MG structure is a tuple* $\mathfrak{A} = \langle \mathscr{A}^{\mathfrak{N}}, \mathscr{A}^{\mathfrak{C}}, D, PF, h\rangle$ *where:*

1. $\mathscr{A}^{\mathfrak{N}} = \langle \mathbb{A}^{\mathfrak{N}}, \leq^{\mathfrak{N}}, \{\otimes_i^{\mathfrak{N}}\}_{i\in I}\rangle$ *is a partially ordered algebra.*
2. $\mathscr{A}^{\mathfrak{C}} = \langle \mathbb{A}^{\mathfrak{C}}, \wedge^{\mathfrak{C}}, \vee^{\mathfrak{C}}, \{\otimes_i^{\mathfrak{C}}\}_{i\in I}\rangle$ *is a complete lattice-ordered algebra of the same type as* $\mathscr{A}^{\mathfrak{N}}$.
3. $D \neq \varnothing$
4. PF *is a set of functions* φ *of type* $D^{\omega} \longrightarrow \mathbb{A}^{\mathfrak{N}}$ *s.t.*
   - *(4.a) For every* 0*-ary function* e *on* $\mathbb{A}^{\mathfrak{N}}$*, there is a* $\varphi_e \in PF$ *such that for any* $f \in D^{\omega}$*,* $\varphi_e f = e$.
   - *(4.b) For every* $m \geq 1$*-ary operation* $\otimes$ *of* $\mathscr{A}^{\mathfrak{N}}$*,* PF *is closed with respect to the type-lifted operation* $\otimes(\varphi_1,\ldots,\varphi_m)f = \otimes(\varphi_1 f,\ldots,\varphi_m f)$.
   - *(4.c) For every* $n \in \omega$ *and* $\varphi \in PF$*, there are functions* $\forall_n, \exists_n \in PF$.
5. $h : \mathbb{A}^{\mathfrak{N}} \longrightarrow \mathbb{A}^{\mathfrak{C}}$ *is such that (where* $\leq^{\mathfrak{C}}$ *is the lattice order):*
   - *(5.a) For all* $a,b \in \mathbb{A}^{\mathfrak{N}}$*,* $a \leq^{\mathfrak{N}} b \iff ha \leq^{\mathfrak{C}} hb$.
   - *(5.b)* $h\otimes^{\mathfrak{N}}(a_1,\ldots,a_m) = \otimes^{\mathfrak{C}}(ha_1,\ldots,ha_m)$ *for every operation* $\otimes$
   - *(5.c)* $h((\forall_n\varphi)f) = \bigvee^{\mathfrak{C}}\{a \in range(h) \mid a \leq^{\mathfrak{C}} \bigwedge^{\mathfrak{C}}\{h(\varphi f') \mid f' \sim_{x_n} f\}\}$

(5.d) $h((\exists_n\varphi)f) = \bigwedge^{\mathfrak{C}}\{a \in range(h) \mid \bigvee^{\mathfrak{C}}\{h(\varphi f') \mid f' \sim_{x_n} f\} \leq^{\mathfrak{C}} a\}$

I'll sometimes refer to $\mathscr{A}^{\mathfrak{N}}$ *as the* nugget *of an MG structure, and* $\mathscr{A}^{\mathfrak{C}}$ *as its* seam.

The idea, then, is to capture the behaviour of Mares-Goldblatt frames by seeing them as embodying *completions* of poset algebras. The nugget behaves like the complex algebra we have in a Mares-Goldblatt frame, with the carrier set as some (perhaps not full) collection of the admissible propositions, and the seam behaves as the full complex algebra of the frame, admitting all propositions. Then we interpret the quantifiers in the nugget by, as it were, "peeking over our shoulder at the seam". This peeking is embodied in conditions (5.c)–(5.d) on h (the order-embedding which completes the nugget), which require that the nugget be such that for any set of 'instances' of $\varphi[x]$, i.e. the set $\{\varphi f' \mid f \sim_{x_n} f\}$, always has an infimum $(\forall_n\varphi)f$ and a supremum $(\exists_n\varphi)f$.

In this talk I'll introduce this algebraic presentation, and prove some basic results concerning it. In particular, I'll compare this approach to interpreting quantifiers to that employed in (Cintula & Noguera, 2021), showing that the MG approach provides a special case of this one – though, as I'll discuss, a special case which provides some reasonable hopes for nice results concerning frame semantics for a range first-order non-classical logic (for instance, such that first-order extensions of the class of logics studied in the context of *gaggle theory* (Bimbó & Dunn, 2008)).

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Embedding Kozen-Tiuryn logic into residuated one-sorted Kleene algebra with tests[1]

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Kozen (1997) introduced Kleene algebra with tests, KAT, as a simple algebraic framework for verifying properties of propositional while programs. KAT is two-sorted, featuring a Boolean algebra of tests embedded into a Kleene algebra of programs.

Kozen et al. (2016) showed that the equational theory of KAT is PSPACE-complete, and Kozen (2000) that KAT subsumes Propositional Hoare logic, PHL. A PHL partial correctness assertion $\{b\}p\{c\}$, meaning that c holds after each terminating execution of program p in a state satisfying b, is represented in KAT by the equation $bp\bar{c} = 0$ or, equivalently, $bp = bpc$.

Kozen and Tiuryn (2003) introduced a substructural logic S that extends KAT and represents partial correctness assertions as implicational formulas $bp \Rightarrow c$. They argue that the implicational rendering of partial correctness assertions has certain advantages over the equational one, e.g., it facilitates a better distinction between local and global properties. PHL embeds into S and S is PSPACE-complete (Kozen, 2003). Kozen et al. (2016) proved that S induces a left-handed KAT structure on programs. However, a more thorough discussion of the relation of S to residuated extensions of Kleene algebra, such as Pratt's (1991) action logic, or to substructural logics in general is not provided. We believe that a deeper investigation of these connections would shed more light on the landscape of program logics and algebras.

We prove an embedding result that might contribute to this. We note that the straightforward extension of KAT with a residual $\rightarrow$ of the Kleene algebra multiplication does not enable an obvious embedding of S: implication formulas of S are test-like in the sense that they entail the multiplicative unit 1, but terms $p \rightarrow b$ of residuated KAT are not test-like, even if b is. We will show that a *codomain*-style operator t together with its adjoint e provide a promising remedy, as $t(p \rightarrow e(b))$ behaves like $p \Rightarrow b$ of S.

Recently, Desharnais et al. (2006) and Desharnais and Struth (2011) studied Kleene algebras with an (anti)codomain operator, KAA, as a one-sorted alternative to KAT. A KAA extends a Kleene algebra $\mathbf{A}$ with a unary *antidomain* operation a, so that $t(x) := a(a(x))$ becomes a *codomain* operation, and $t(A) = \{x \in A : t(x) = x\}$ becomes the universe of a Boolean subalgebra of $\mathbf{A}$ in which $a(x)$ is the complement of x in $t(A)$. We will, however, not need the full power of KAA. We will exhibit a one-sorted generalization of Kleene algebras with (co)domain called OneKAT, into which the equational theory of KAT embeds, yet is a conservative extension of Kleene algebras (which KAA is not). We shall show that if OneKAT is extended with residuals, one can define an anticodomain operation as $t(t(x) \rightarrow 0)$. We will then further extend OneKAT by adding the adjoint e of t to obtain a variety called SKAT.

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Our main technical result is that S embeds into the equational theory of SKAT^*, the class of algebras in SKAT that are based on *-continuous Kleene algebras.

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Paradox and substructurality

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I start by recalling three of the main paradoxes in philosophy of logic: the paradoxes of *selfreference*, the paradoxes of *vagueness* and the paradoxes of *implication*. I introduce the notion of a *substructural* logic and the main structural principles (*reflexivity*, *monotonicity*, *transitivity*, *contraction* and *commutativity*), for then presenting the main *advantages* that approaches to those paradoxes that use a substructural logic enjoy over approaches that use a structural (and nonclassical) logic. On this basis, I present the *standard understanding of substructurality*, according to which *the structural level is prior to the level of the logical operations* (*conjunction*, *disjunction*, *negation*, *implication etc.*, see Došen, 1989). Relatedly, I present a tempting *explanation of the advantages of substructural approaches to paradox* that builds precisely on such understanding, by claiming that those advantages accrue to substructural approaches *exactly because substructural logics operate at a more fundamental level than structural logics do* (Ripley, 2015).

With this background, I propose and explore an opposite understanding of substructurality, according to which *it is the operational level that is prior to the structural level*. On such understanding, a logic's denial of a classical structural principle is due to its denial of some classical principle concerning conjunction, disjunction or implication. The main argument I develop in favour of this hypothesis seeks to establish the conclusion that *the main components of the structure of logical consequence* (premise combination, conclusion combination and entailment) *consist in certain* (conjunction-like, disjunction-like and implication-like respectively) *logical operations that are naturally definable in the broad framework of the target logic* (although, of course, they need not be expressed by any particular operator that may be present in the specific language that is used in a certain presentation of the logic, even if that presentation has been historically salient in the development of the logic). For example, the premise combination A,B has the force of representing A and B as *holding together*, which, presumably, is tantamount to representing that A holds (*i.e.* is true) *and* B holds: therefore, premise combination would seem to consist in a certain kind of conjunction.

Additionally, I develop three further, auxiliary arguments for the conclusion that the main components of the structure of logical consequence consist in certain logical operations. The first auxiliary argument is that there are, for example, *different types* of premise combination, and that such difference is very naturally accounted for in terms of different logical operations (e.g. intensional *vs* extensional conjunction). The second auxiliary argument is that the conclusion offers *the best explanation* for the remarkable correlation between, for example, premise combination and conjunction (*i.e.* the fact that A,B entails C iff $A\&B$ entails C). The third auxiliary argument is that, in the *semantics* of many substructural logics, a logical consequence to the effect that the premises $A_1,A_2,A_3,\ldots,A_i$ entail the conclusions $B_1,B_2,B_3,\ldots,B_j$ is defined as the logical truth of the implication $(A_1\&A_2\&A_3\ldots\&A_i)\rightarrow(B_1\vee B_2\vee B_3\ldots\vee B_j)$, so that, in such logics, premise combination, conclusion combination and entailment are wholly understood in terms of the logical operations of conjunction, disjunction and implication respectively. (I emphasise that all these arguments rely on assumptions about the target logic that, while almost always unquestionable for virtually all the logics of interest for this talk (and for many structural logics including classical logic), might not be such for other (substructural or structural) logics, and the following conclusions about "logics" should accordingly be understood as implicitly so qualified.)

On this basis, I then claim that one can understand the fact that certain structural principles hold or do not hold in a logic *as the result of the fact that the corresponding principles for conjunction, disjunction or implication hold or do not hold in the logic*. For one example, monotonicity holds in classical logic but not in nonmonotonic logics because $A\&B$ entails A in classical logic but not in nonmonotonic logics. For another example, transitivity holds in classical logic but not in nontransitive logics because $A \rightarrow B$ and $B \rightarrow C$ entail $A \rightarrow C$ in classical logic but not in nontransitive logics. For yet another example, contraction holds in classical logic but not in noncontractive logics because A entails $A\&A$ in classical logic but not in noncontractive logics. Assuming that this is right, I submit that it implies the need for a *reconceptualisation of substructural logics*, not as logics that fundamentally deny some structural principle of classical logic, but as logics that *fundamentally deny some principle of a certain specific kind that conjunction, disjunction or implication obey in classical logic*—that is, the kind of principles that determine that classical logic has the structural principles it has. That arguably does make substructural logics less categorically different from structural nonclassical logics than is commonly assumed: they all *fundamentally deny some principle of the logical operations*. The real difference is in that they centre on logical operations *other than negation*. Indeed, they typically *do not centre on implication either* and thus centre on logical operations (*i.e.* conjunction and disjunction) all of whose argument places are *upwards monotonic*—that is, in effect, logical operations of *positive composition*. Such centring is clear for nonmonotonic, noncontractive and noncommutative logics, but can also be argued to be there for at least a certain class of nontransitive logics.

I conclude that substructural approaches to a paradox thus typically individuate the *crux* of the paradox in a *peculiar behaviour of positive composition*. Such a take on a paradox might initially come across as rather surprising and unlikely given the feeling of familiarity and obviousness that positive composition emanates as opposed to other kinds of logical operations. However, importantly, even with substructural logics so reconceptualised, substructural approaches to paradox retain all the advantages mentioned in the first paragraph. That can then be understood as evidence for the idea that the paradoxes in question are indeed rooted in mistakes that we're led to make *when (explicitly or implicitly) operating with conjunction and disjunction in the course of a paradoxical reasoning*.

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